A Genetic Algorithm with Monte-Carlo Simulation for an Optimal Inspection Allocation in a Batch Assembly Line with Tolerance Stack-up

by

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A Thesis
presented to
The University of Guelph

In partial fulfilment of requirements
for the degree of
Master of Applied Science
in
Engineering

Guelph, Ontario, Canada

ABSTRACT

A GENETIC ALGORITHM WITH MONTE-CARLO SIMULATION FOR AN OPTIMAL INSPECTION ALLOCATION IN A BATCH ASSEMBLY LINE WITH TOLERANCE STACK-UP

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In this thesis, the total inspection policy cost of a batch assembly line in the multi-stage production system (MSPS) is optimized by using the Genetic Algorithm with Monte Carlo simulation. Total inspection policy cost (consist of inspection, rework and penalty costs) can be optimized by allocating different inspection strategies without compromising the quality of the final product. Inspection (Full, Sample, No inspection) is allocated at each station in such a way as to reduce the total inspection policy cost. As far as concerning tolerance stack-up which mainly depends on optimizing the limits (lower and upper inspection limits), the Genetic Algorithm is trying to optimize the limits as closely as possible to reduce the penalty cost. In multi quality characteristics problem if the part fails in any of the impacted quality characteristics, it goes for rework and reworks cost is added depends on part fails in which quality characteristics.

Dedicated to my beloved family members,

my grand parents Mr. & Mrs. N.B. Patel

my parents Mr. & Mrs. P.N. Patel

my sister Khushbu

&

my loving wife Shweta

ACKNOWLEDGEMENTS

Throughout my thesis writing, firstly I would like to express my sincere gratitude to my advisor Dr. Fantahun Defersha for his continuous encouragement and support during my MASc study and related research. The professor guides me at each level of research through the formulation of the research topic to methodology implementation. Without his support, it would be a very difficult task for me to complete it. I can not imagine the completion of my thesis without his guidance. Thank you so much for being my advisor.

I would like to thank my advisory committee member, Dr. Ibrahim Deiab, for his insightful comments and encouragement. Also, I would like to thank my family members and especially my grandfather for giving support in every decision of my life and always there for me. Finally, I would like to thank my wife who always stay with me and believe in me.

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LIST OF SYMBOLS

σ	Standard Deviation
σ^2	variance
μ	Mean
C_p	Capability index

LIST OF ACRONYMS

MSPS	Multi Stage Production System
BP	Batch Production
EA	Evolutionary Algorithm
GA	Genetic Algorithm
MCS	Monte Carlo Simulation
SA	Simulated Annealing
COQ	Cost Of Quality
QFD	Quality Function Deployment
U	Upper specification limit for tolerance
L	Lower specification limit for tolerance
S	Sample inspection
F	Full inspection
N	No inspection
PDF	Probability Distribution Function
CDF	Cumulative Distribution Function
RSS	Root Sum Squares
ppm	Parts Per Million
LIL	Lower Inspection Limit
UIL	Upper Inspection Limit

Chapter 1

Introduction

1.1. Chapter Outline

The main purpose of this introductory chapter is to provide details about the different terminologies of quality and understanding of inspection policies. Different models are used for achieving highest quality in manufacturing industries. This chapter contains different quality definitions which can be found in published textbooks or research papers on specific subjects in-depth but some of the terminologies that are used in this thesis explained in-depth as this thesis as well. Readers can also refer standard books and research articles for a deep knowledge on specific terminologies.

After general introduction on quality in Section 1.1, a brief discussion about quality control and definitions of quality are explained in section 1.2. Different quality models and cost optimization techniques are discussed in this section. Also, overview on quality control of manufacturing assembly line is explained in brief. Quality function deployment and house of quality are explained in section 1.3. Different inspection allocation strategies are explained in section 1.4.

Discussion on tolerances, their effects on quality and different models which are used to calculate the tolerance stack-up are explained in Section 1.5. Examples of production line are given which works on quality criteria which includes 6σ quality control. A brief discussion about Monte Carlo Simulation(MCS) is explained in section 1.6 and Genetic Algorithm(GA) is explained in Section 1.7.

1.2. History and importance of quality

Quality can have several meanings, but if we have to give the simple ideas about it then we can say that a it refers to better fulfilment of standard requirement as measured against a similar kind of other things. The quality movement has its roots for many decades. As considered in the way of industrial purpose then it was started in Europe around 13th century, where artisans began to make unions called guilds. The role of these guilds was to develop strict rules and regulation for product and service. After that, inspection committees implemented those rules and regulations by marking perfect goods with a special symbol. In the beginning, artisans used to put the mark on their products. At that time, it was tracked as the origin of product. But after that implementing the inspection rules and guidelines, that mark was known as the symbol of good quality. Moreover, this approach had made industrial revolutionary moment during the 19th century for manufacturing quality products. During the factory system, supervisor was appointed on the labours for the inspection of quality of product. And if defective products were found, they sent defected products for rework or in scrap. By this way, they were not losing their customers. During the world war, armed forces did not compromise the quality of the weapons, so at that time for the fast production without compromising quality, they introduced sampling inspection. Arm Forces adopted Walter Shewhart's statistical quality control techniques and strictly followed the Mil-Std-105 standard. Two American experts of quality control named, W. Edwards Deming and Joseph M. Juran (Known as Father of Modern-day Quality Management) gave their contribution in Japan's goods quality, and their efforts are seen everywhere in the world that Japanese goods are famous for their quality products.

1.2.1. Definitions - Quality

Quality can be defined in many ways. It can be defined as per its categories. The quality definition can be described by five approaches as per the perspective of Garvin,D.A. (Managing Quality)(1988).

1. **Product based quality definition:**

   In this term, quality is measured by precise measurement of variable or product, which means differences in quality shows differences in the quantity of desired product (L.abbot).

2. **Transcendent quality definition:**

   This perspective deals with the hypothetical characteristic of quality. In which quality can be defined as something toward which we struggle as a standard, but may never contrivance completely.

3. **User-Based quality definition:**

   In User-based perspective, quality can be as per the market-oriented or customer-oriented. Meanwhile it fulfills the customer needs and fit for the use. Quality can be customized as per the requirement of customer.

4. **Manufacturing based quality definition:**

   This perspective represents quality as per standard requirements. In this aspect of quality,products have to fulfill the standards such as ISO 9001 for manufacturing products.

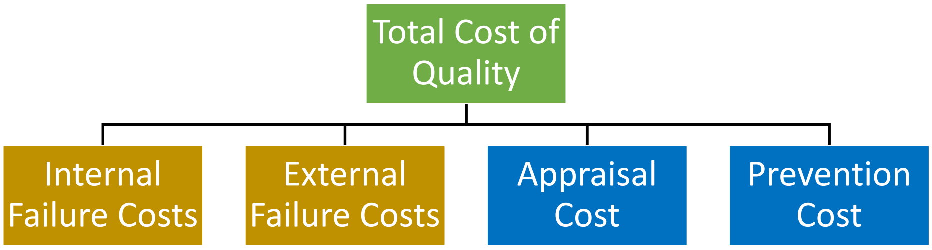

Figure 1.1: Cost of Quality

5. **Value-based quality definition:**

   This perspective of quality is cost-oriented which can satisfy the demand of customers as they can get product at affordable cost and also satisfy the suppliers, as they can produce the product at acceptable cost.

The main concern of this thesis is to optimize the test cost by applying different inspection strategies, for that, value-based quality is more acceptable.

1.2.2. Cost of Quality

Cost of Quality (COQ) can be defined as to get high quality products by adopting a certain methodology which can be responsible for the result of organization's internal or external failure. There are main four objective which affect on Cost of Quality which is shown in fig.1.1.

1. **Prevention Cost:**

This cost is associated with designing, planning, maintenance, training of worker of quality management.

2. **Appraisal Cost:**

   This cost is mainly associated with measuring, monitoring, auditing of quality of product as per customer requirement.

3. **Internal Failure Cost:**

   This cost is associated with delivering the quality products to the customers and during that process, defected parts or products can be reworked or scraped which cost to company. This cost is prevented inside the company, so it does not affect ratings of customers.

4. **External Failure Cost:**

   This cost is associated with the product which is delivered to customer and customer will find defected product. This cost contains penalty cost, repairing cost, return cost. It also affects the company's ratings.

In this thesis, the main focus is to reduce the external failure cost by delivering high quality products. Also reduce the internal failure cost by obtaining optimum solution.

1.3. QFD - Quality Function Deployment

It is identified as customers' requirements means "What" is desired by a customer in market. Moreover, to complete this desire, the next step is "How" it can be possible, which means by implementing which kind of engineering process is required for that. This both condition "What" and " How" correlate with each other.

House of Quality:

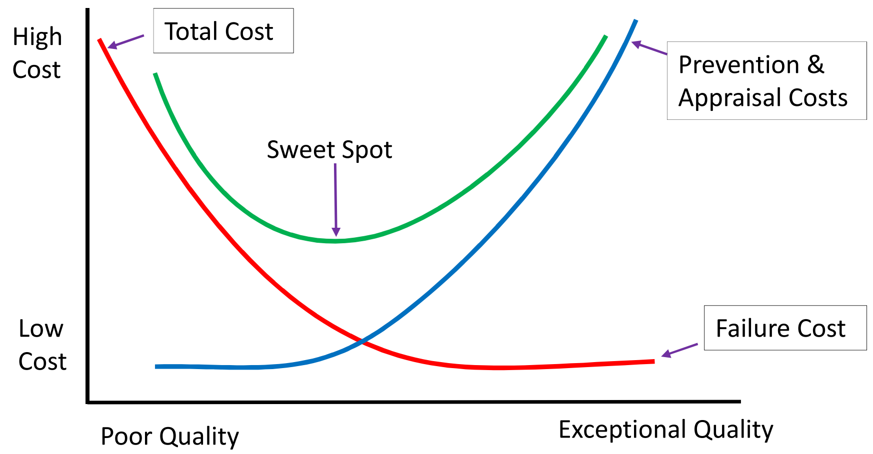

Figure 1.2: Classical model of optimum quality cost
From Jurans Quality Control Handbook, 4th edition. J.M. Juran, editor. Copyright © 1988, McGraw-Hill

It is part of QFD. It was first appeared in Mitsubishi Heavy Industries for designing oil tanker in 1972. House of Matrix in nothing but a tool of QFD which shows by matrix. This matrix can be explained step by step by taking simple example.

Example: (Let us take a simple example of Burger)

1. **Identify the customer requirements**

   (Means which kind if Burger is required by the customers from their perspectives like good taste, low price, low in fat, good texture, fresh and hot, grilled, etc. These all features are identified as customer requirements.)

2. **Identify, the product can fulfill customer requirements**

   (Means How can company satisfy customers requirements like increase the toppings, low-fat cheese, different sizes, etc.)

3. **Identify, How's relationship**

   (Means How will customers satisfy by product)

4. **Ratings**

   (By taking the ratings from a customer for company's different approach to satisfy their demand. So from rating, company can go towards its decision for improvement in particular area)

5. **Evaluate services and product satisfaction**

   (In this step, Company compare their features with its rival company's product)

6. **Determine Technical Attributes**

   (In this step, checking the performance by determining the technical attributes)

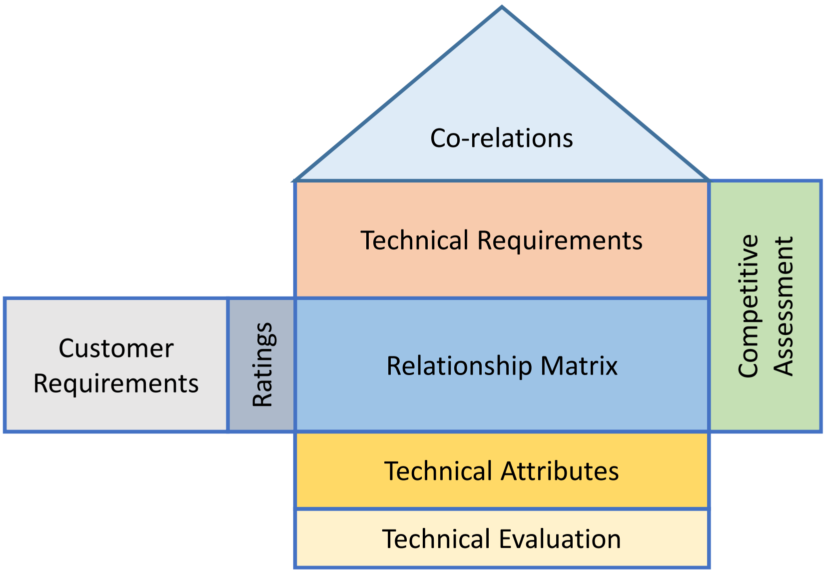

Figure 1.3: House of Quality - QFD
from https://www.conceptdraw.com/solution-park/house-of-quality

1.4. Inspection:

According to Menipaz (1978) ,Inspection can be described by two main strategies.

1. **Sorting :**

   In the Sorting or Sampling Process, Good and defective parts are sorted by sampling process to achieve quality

2. **Process Control or Quality control :**

   In the process control, for obtaining high quality, continuous inspection or monitoring is required on process.

Inspection is used as a part of quality control. Inspection is carried out during the manufacturing of goods to check the quality. If the parts are bad in quality then it will be gone for scrap or rework. If number of defected parts are more, manufacturer may take the decision to reject whole batch. Inspection is useful for :

- Differentiate good and bad parts
- Quality measurement (high to low quality)
- Measuring of accuracy instruments
- Securing product

In this thesis, the batch inspection is allocated by three types of inspection policy.

1. **Full Inspection** (F)
2. **Sample Inspection** (S)
3. **No Inspection** (O)

Full inspection is preferable if the quality must not be compromised. However, by allocating full inspection, it will increase the inspection cost, and it directly affects on overall product cost. Sample inspection can be implemented in the criteria where less inspection is required. In some cases, if the testing is destructive, it is necessary to apply sample inspection. In some conditions, no inspection is implemented where inspection is not necessary. For the high quality products, it is always preferable to allocate full inspection. If Sampling Inspection is allocated, it would be the possibility to pass defective parts to customer because in sample inspection, some of the samples are inspected. Sample inspection may reduce the inspection cost but may possible to increase the penalty cost.

1.4.1. Sampling Inspection

In the inspection procedure, sampling inspection is the optimal strategy because full and no Inspection has their fixed cost, but only sample inspection can be varied by applying different sample size (s_i) and acceptance number (a_i) which can be affected on cost directly. For better understanding, let us take an example. Assume one company is produced 1000 parts per day. If sampling inspection is allocated for quality, sample size (s_i) is taken as 100. While inspecting the parts, mostly 2 to 3 parts are found defected in one sample. So quality inspector has decided the acceptance number (a_i) for sample size. Let's take $a_i = 3$ in this case. Means, in any samples, if quality inspector finds more than 3 defected parts then whole batch is rejected. Otherwise only defected parts are going for rework. So sample size and acceptance number are most effective variables in quality control.

1.5. Tolerance

Tolerance design is considered as a very complex issue whenever concerning about the quality products for any manufacturing industries. Tolerance stack-up is

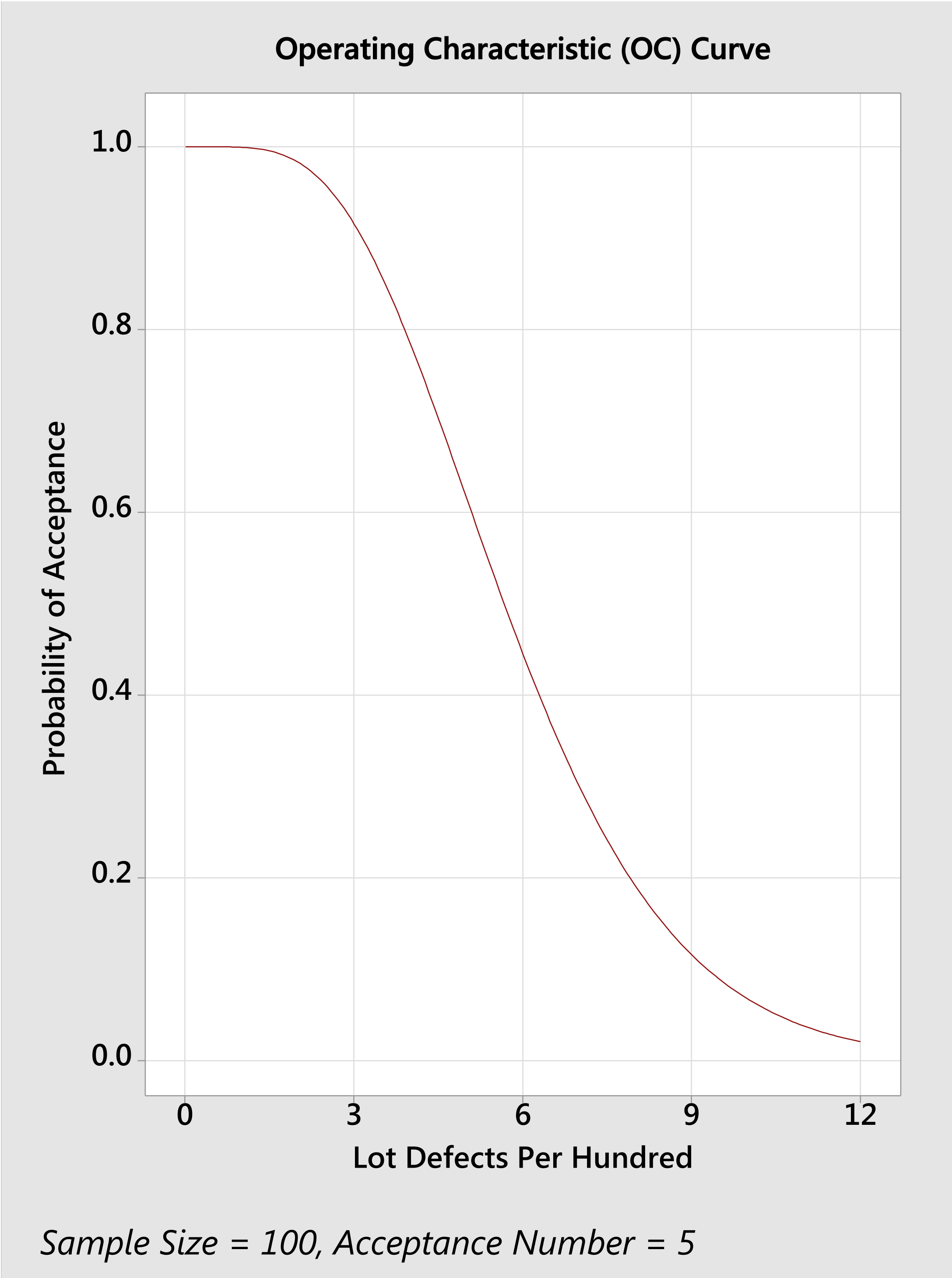

Figure 1.4: Operating Characteristics Curve

played vital role during the assembly of mechanical parts as well as cost optimization. Calculation of optimized tolerance for reducing cost was introduced by Speckhart (1972) and allocation of tolerance was introduced by Spotts (1973). Ostwald and Huang (1977) used to achieve optimal tolerance allocation by using linear programming, which was suitable for sequences and tolerance operation. After that, W J Lee (1989) introduced new approach, a discrete cost—tolerance model using programming. W. H. Greenwood (1988) presented the reciprocal model for consideration of discrete as well as continuous cost function with better empirical data fitting capability.Chas K.W and L.F (1990) introduced quadratic programming method which can solve the sequence problems Zhang and Wang (1993), introduced first-time nontraditional technique (Simulation Annealing) which was mathematical model and can solve continuous cost function for manufacturing process. Iannuzzi and Sandgren (1995) offered Monte Carlo Simulation based tolerance analysis by using genetic algorithm to determine maximum tolerance zone value.M.D. Al-Ansary (1997) implemented model which was solved by worst-case tolerance stack-up criteria by using genetic algorithm.

1.5.1. Methods to calculate tolerance stack-up

Different methods are used to describe tolerance stack-up by sahani A.K. Sahani (2017).

- Worst-case Analysis
- Statistical Tolerance Analysis
- Monte Carlo Simulation

1.5.2. Worst Case Analysis

This Arithmetic method is also known as linear stack-up or maximum-minimum calculation method. This method is used to calculate individual tolerance effects on whole assembly by using upper and lower limit sizes. There are some limitations of this method like when the individual parts in the assembly are increased, the accuracy of worst case method is decreased. So it is suitable for small assembly. Moreover, this approach is suitable, where 100% acceptance mandatory. To determine the mean value(μ) for the component (Mechanical, industrial and technical calculations)

$$\mu = \sum_{i=1}^{k-1} \mu_i - \sum_{i=k}^{n} \mu_i; \tag{1.1}$$

to calculate tolerance (T) for component,

$$T = \sum_{i=1}^{n} T_i; \tag{1.2}$$

where,

k = Number of components;

T_i = Tolerance of component

μ_i = Mean value of component

n = Number of partial components

1.5.3. Statistical Tolerance Analysis

This tolerance analysis method uses the probability distribution function (PDF). This method gives the partial interchangeability of assembly without spoilage of favorable cases. As a result, it gives lower manufacturing cost concerning larger tolerances. This method is used in mass production where optimization of manufacturing cost is required most. The close tolerance variation can be described by Gauss curve of probability density. Most of the cases, frequency of occurrence for individual component dimension matches with **Normal Distribution**. The Gaussian curve for normal distribution having different mean (μ) and standard deviation (σ) value is shown in fig.1.5.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{x-\mu^2}{2\sigma^2}}; \tag{1.3}$$

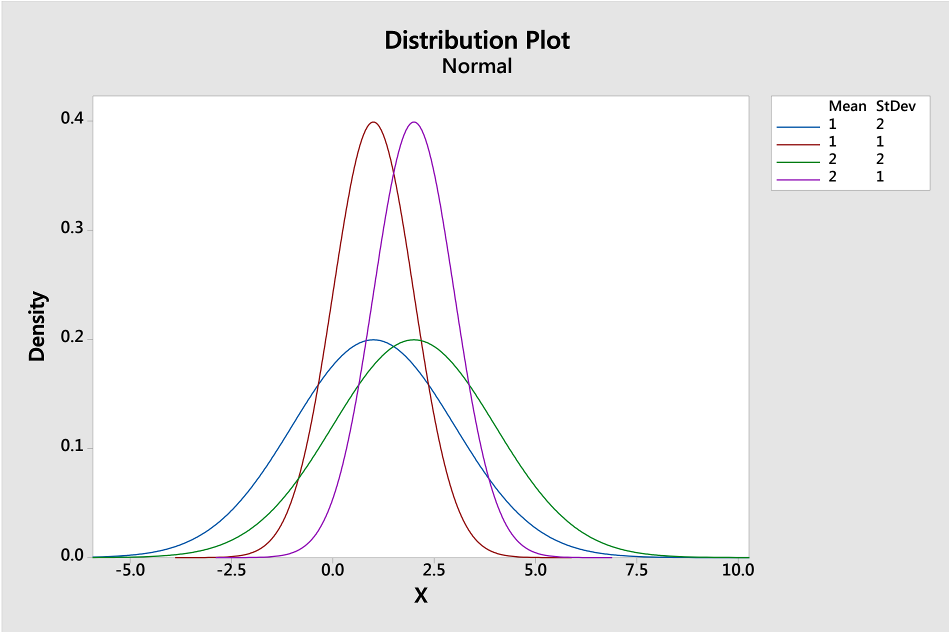

Figure 1.5: Gaussian or Normal distribution curve

where σ = Standard Deviation

RSS (Root Sum Squares) Method :

RSS method is considered for individual component manufactured with assumption of level of process capability of 3σ. So the tolerance interval of $\mu \pm 3\sigma$ is shown in fig.1.7,

For the single quality characteristic of multi-stage process Sofie Van Volsem (2007), assembly components must be ($k \geq 2$) considered.

Mean μ_i is the midpoint of the tolerance region, which can be shown in fig.1.6 for linear function.

$$\mu_i = \frac{U_i + L_i}{2}; \tag{1.4}$$

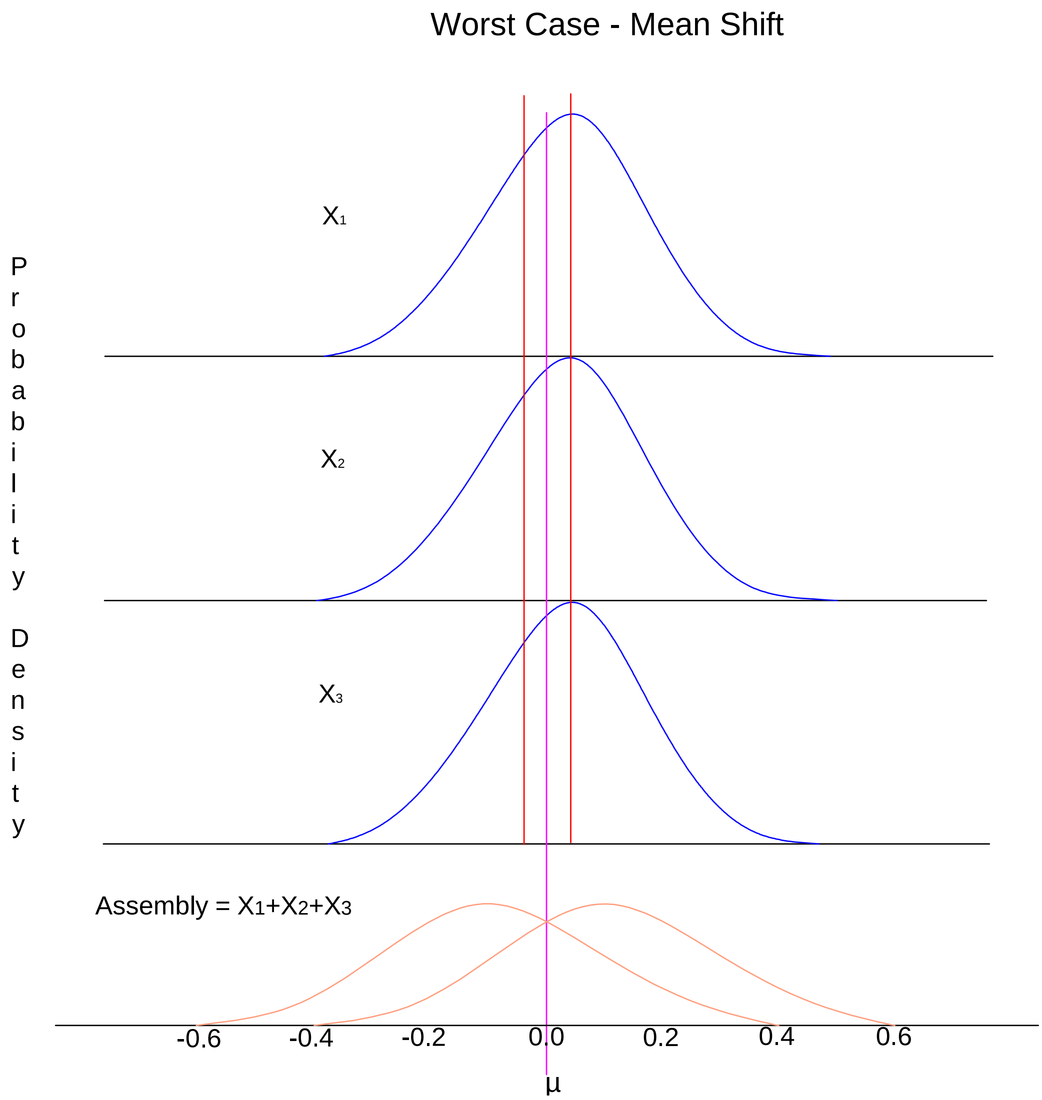

Figure 1.6: Worst case analysis - Mean shift

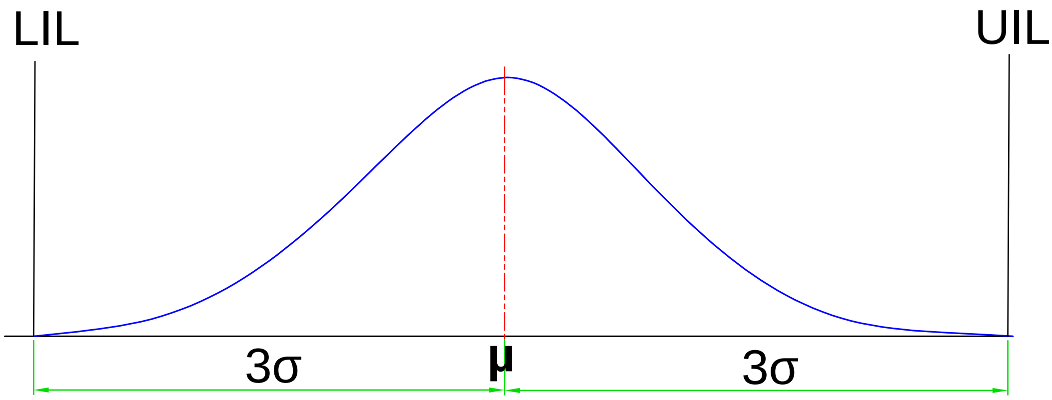

Figure 1.7: RSS analysis (Courtesy MIT Calc.com)

1.5.4. Six Sigma Method :

However, 3σ analysis is not sufficient for the highest quality approach. As shown in table 1.1, different values of mean shift and standard deviation are shown. It is clearly shown that by using 3σ method, total production scrap is 2700 per million productions which is no small amount. Also from fig. 1.9, a graph shows difference of $+1.5\sigma$. Which means it is not possible all the time to fix the value of mean value in the center of the tolerance. From the research, it is assumed that $+1.5\sigma$ deviation occurs at that time production scrap is increased 2700 to 66803, which is loss of money. If the graph is extended to the 6σ then the loss can be minimum, i.e. approximately 3.4 per million productions are shown in fig. 1.9, which means if the quality control has adopted full inspection allocation then no defected parts can be delivered to customers. Six sigma method was first introduced by the Motorola Company Pyzdek (2003). The well-known example of six sigma method is used in Toyota Motors Manufacturing Company. Company

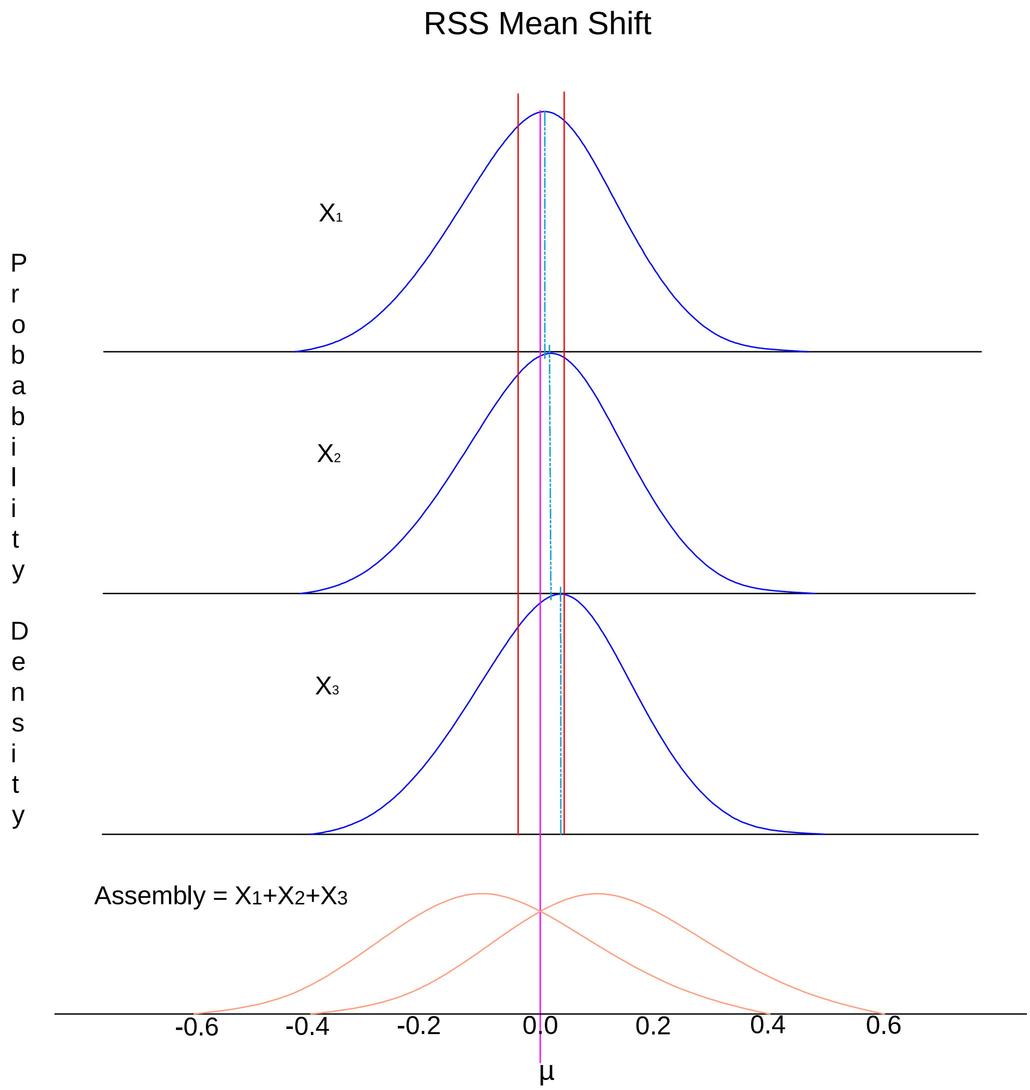

Figure 1.8: RSS Analysis - Mean shift

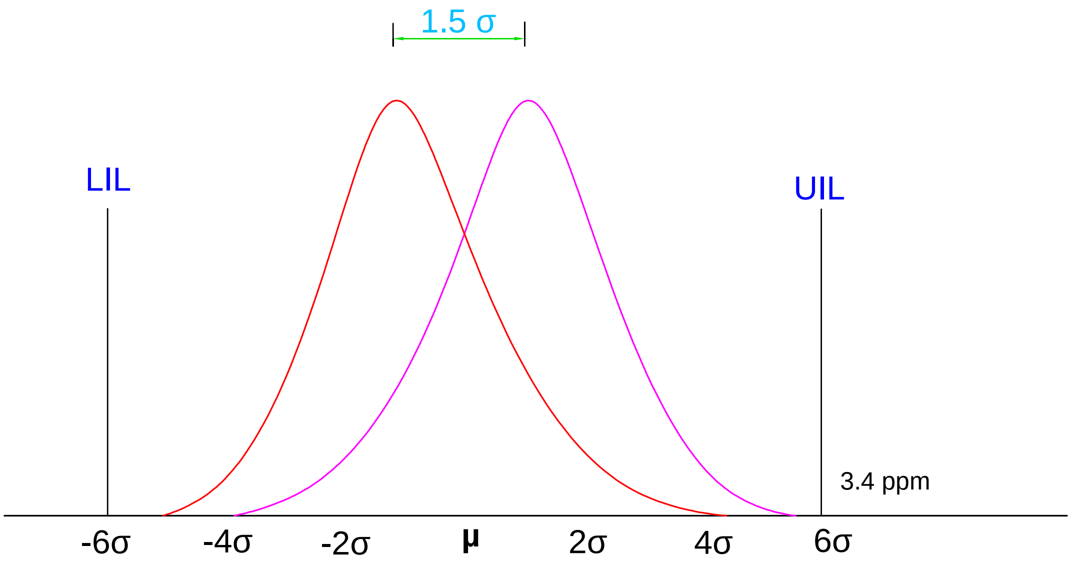

Figure 1.9: Mean Shift

strictly follows the six sigma policy for mass production of cars and reduce the rate of scrap.

Table 1.1: Mean Value and Deviation value comparison

Tolerance Limit	Yield point	No. of Rejected parts per million production
$\mu \pm 1$	68.2	317310
$\mu \pm 2$	95.4	45500
$\mu \pm 3$	99.73	2700
$\mu \pm 4$	99.994	63
$\mu \pm 4.5$	99.9993	6.8
$\mu \pm 6$	99.9999998	0.002

From table 1.1, the defected parts reached the customer are drastically decreased, but the cost of inspection is increased as the full inspection required for each assembly. Another way to reduce the defective parts reach to customer is to decrease the gap between the lower inspection limit and upper inspection limit.

As this limit is going to narrow, automatically defected parts are increased during the inspection and going for scrap or rework. By tightening the limits, the defected parts are increased drastically, which is also cost to company. So optimum strategy should be allocated so the waste should be minimum, and customer can get zero defected parts. For that, impressive inspection allocation strategy is required by applying sampling inspection, no inspection wherever required, which can reduce the inspection cost.

Process Capability Index

The process capability index C_p is the ratio of measurement of the capability of a process to manufacture a product that encounters its dimensions. Process capability is used for

- reviewing of tolerance design
- calculating the process yield
- process auditing
- measuring the effect of new developing or changing the process

Process Capability Index C_p is calculated by

$$C_p = \frac{U - L}{6\sigma}; \tag{1.5}$$

where,

U = Upper specification limit for tolerance

L = Lower specification limit for tolerance

σ = Standard deviation

Mostly the value of $C_p = 1$ so the tolerance spread equally for 3σ model. Here, value of C_p can be varied according to design specification. Like in the previous example, if the value of $C_p = 1$ for 3σ model then defective parts may produce approximately 2700 per million productions. But if value of $C_p = 1.33$

than defect rate may approximately 63 ppm. So the difference is appreciable by changing the value of C_p. The different values of C_p and approximate defected parts related to that value is shown in table1.2. Different values of C_p is also shown in fig. 1.12.

Table 1.2: Process capability Index (C_p)

Cp	ppm (Parts per Million)
0.33	317,311
0.67	45,500
1.00	2,700
1.10	967
1.20	318
1.30	96
1.33	63
1.40	27
1.50	6.8
1.60	1.6
1.67	0.57
1.80	0.0067
2.00	0.002

The C_{pk} Index

The capability index C_{pk} can be measured as the distance between the mean value μ to the nearest tolerance limit which ever $(U or L)$ with respect to 3σ. Mostly the value of C_p and C_{pk} are same when data is centered otherwise it may vary. The value of C_{pk}may vary between 0.135% to 0.27%. C_{pk} can be found by

$$C_{pk} = \min(\frac{U - \mu}{3\sigma}, \frac{\mu - L}{3\sigma}) \tag{1.6}$$

1.6. Monte Carlo Simulation

Monte Carlo Simulation is named from famous city of Monaco, which is famous for gambling. So this simulation is as per its name because it is based on randomness. It is used to generate random variables for analysis in various field for

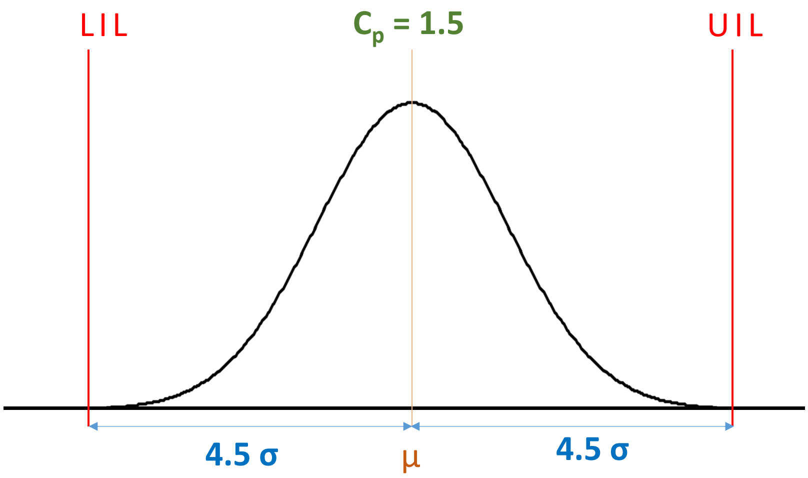

Figure 1.10: C_p Comparison with respect to mean shift

nonlinear engineering models. This technique is important for sampling. The first time, it was used to solve the complex problem of neutron diffusion in 1944. Monte Carlo Simulation (MCS)is based on random sampling for that large number of experiments are conducted on computer. After that, all the statistical characteristics from reports of that experiments are observed. Then after, final Model is drawn from statistical experiments. Random variables are generated as per the distribution (Normal, Uniform, Exponential, etc.).

General steps for MCS are shown in fig. 1.14.

- **Generate Random Variables**

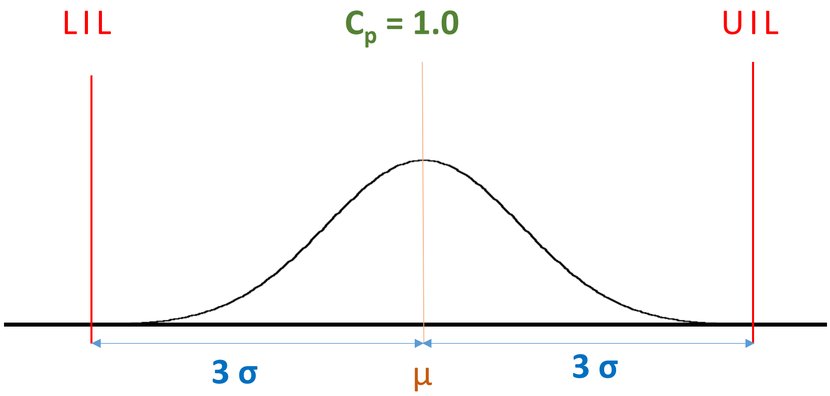

Figure 1.11: C_p Comparison with respect to mean shift

Random number is generated between [0,1] uniformly. Then they can transform as per the distribution into real numbers. In early era, random numbers were generated by throwing dice or many other ways, but now it can be generated by the computer. Here one of the examples of random numbers is shown in table 1.3.

- **Evaluating a model**

  After generating uniform random numbers, the next step is to transform random numbers into samples of uniform variables. Let us take as $Z = (Z_1, Z_2, Z_3, \cdots, Z_N)$ where N = No. of samples generated, The most common and straightforward transformation method is inverse transformation method.

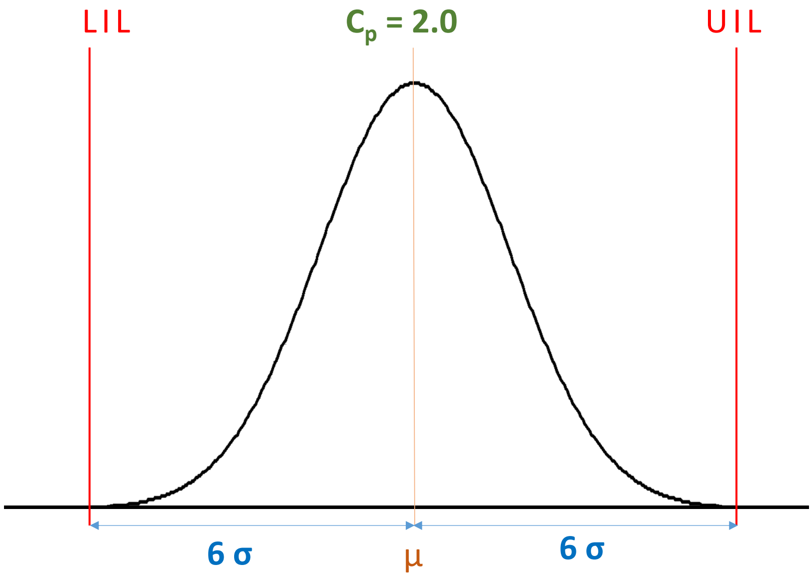

Figure 1.12: C_p Comparison with respect to mean shift

Table 1.3: Random Number Generation

Sr. No	Random Numbers
1	0.3173
2	0.5721
3	0.0785
4	0.0552
5	0.3296
6	0.6613
7	0.4591
8	0.0634
9	0.4721
10	0.4657

$$X_i = {F_{X_i}}^{-1}(Z_i) \tag{1.7}$$

where $i = 1, 2, 3, \cdots, N$

Let's consider Normal Distribution$N(\mu_x, \sigma_x)$; (μ = Mean and σ = Standard deviation)

$$Z = F_X(x) = \phi(\frac{x - \mu_x}{\sigma_x}) \tag{1.8}$$

Here, N samples are generated for each random variables and all N sets have been stored in output model $Y = g(X)$. Problem is solved up to N times.

$$y_i = g(x_i) \tag{1.9}$$

where $i = 1, 2, 3, \cdots, N$

- **Analysis of Output (Extracting Probability Outcome)**

  As in the previous step, Y is obtained, which represents the output of N samples. Different characteristics from output (Y) can be estimated like PDF to CDF, mean(μ), standard deviation (σ).

  Mean can be calculated by

$$Y = \frac{1}{N}\sum_{i=1}^{N} y_i \tag{1.10}$$

  For Variance,

$$\sigma_Y{}^2 = \frac{1}{N-1}\sum_{i=1}^{N} (y_i - Y)^2 \tag{1.11}$$

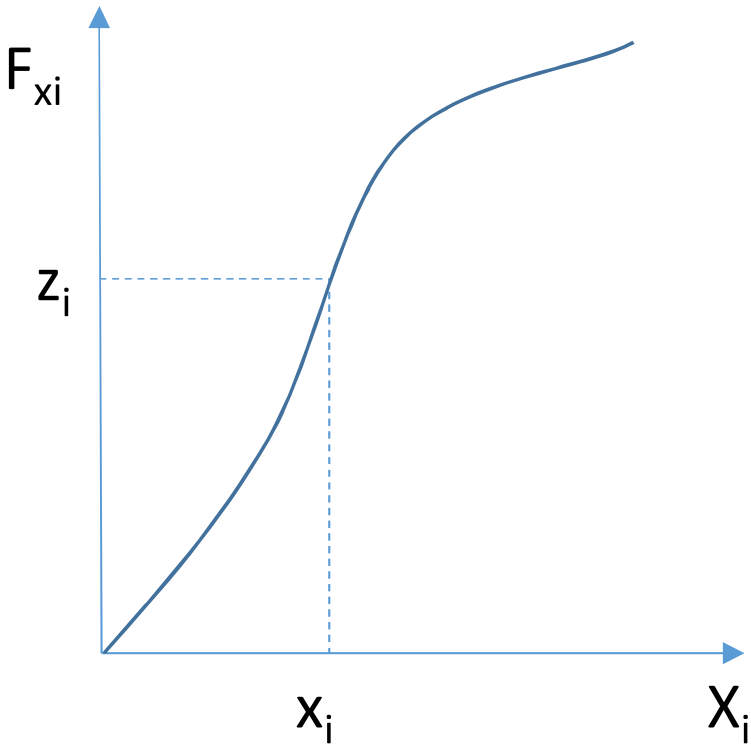

Figure 1.13: Inverse transformation

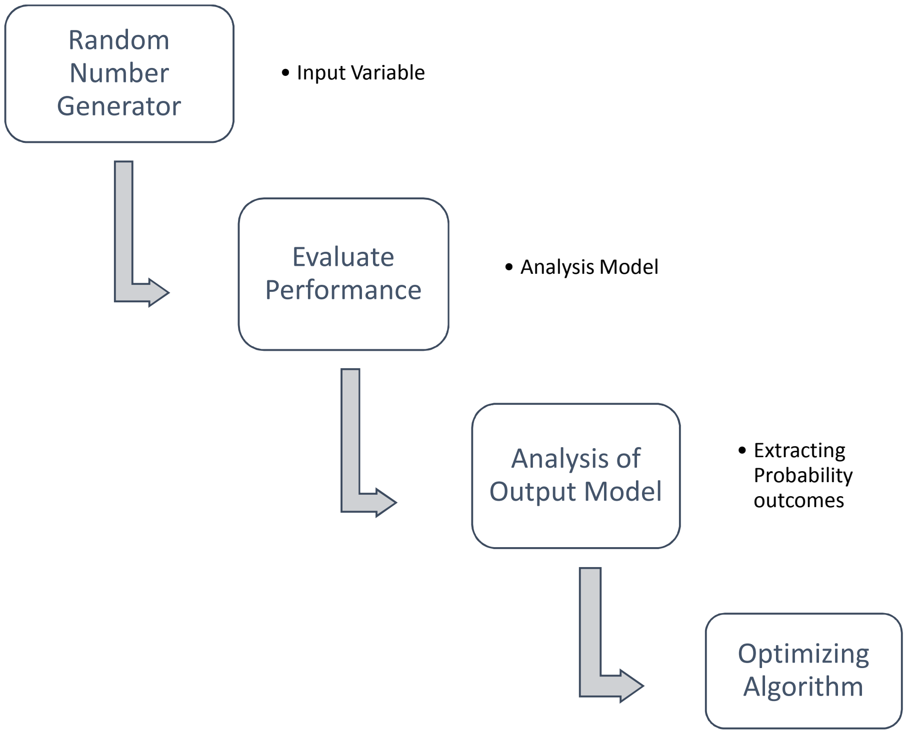

Figure 1.14: Monte Carlo Simulation steps

1.7. Genetic Algorithm

What is the Genetic Algorithm (GA)

GA is an adaptive experiential search algorithm which is based on evolutionary ideas of natural selection. It is inspired by Darwin's theory of "Survival of the fittest." GA is a part of evolutionary computing, a rapidly growing area of artificial intelligence. GA represents an intelligent utilization by using randomness search to solve optimization problems. Genetic Algorithm (GA) is a potent tool for optimization. This approach is the combination of probabilistic selection and direct method, which makes it robustness and flexible system. Some features of GA which can defer it from other optimization techniques are as below.

Introduction to GA:

In computer science and AI, there is a process of search through the space of possible solutions. The set of possible solutions defines the search space (also called state space) for a given problem. Solutions or partial solutions are viewed as points in the search space. In engineering and mathematics, as a process of optimization, the problems are first formulated as mathematical expression. Then for finding a optimal solution, discovering the parameters that optimize the model or the functional components.

- Probability transition rules used instead of deterministic rules.
- Instead of using derivative information, it uses objective function information
- It works on coding.

Genetic Algorithm (GA) is followed by steps shown in Fig 1.15 from Shukur *et al.* (2015).

Genetic algorithm is used to solve optimization problems for constrained

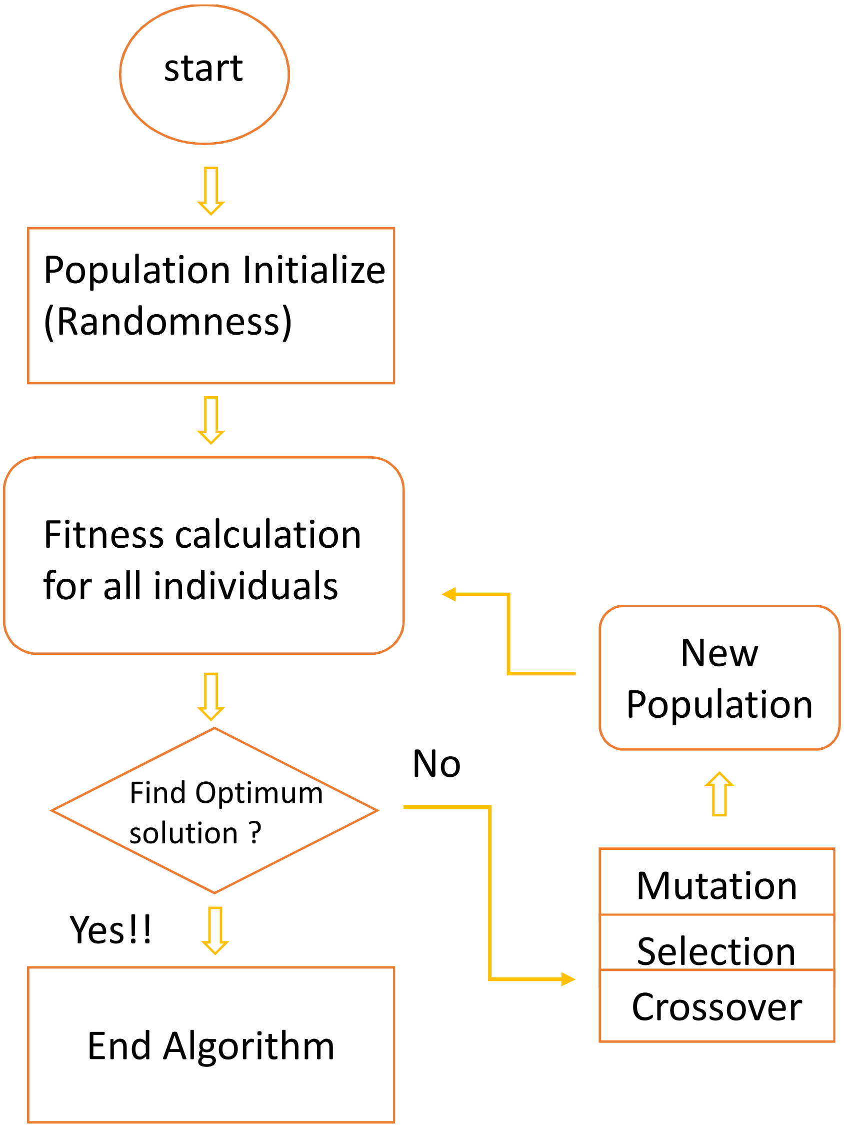

Figure 1.15: Genetic Algorithm steps

and unconstrained scenario based on natural selection. GA is a biological evolution, which continuously modifies the population of individual solution. In GA, individuals are selected on random basis from the current population, which are parents who produce children for next generation. After generating successive population, it evolves to find optimum solution.

There are three main procedures for Genetic Algorithm to create next generation:

- **Selection :** In this process, Individuals are selected from the current population (which are called Parents) and give contribution to generating next generation or population.
- **Crossover :** In this process, Parents, are making the pair to form children for next generation.
- **Mutation:** In this process, individual are changed randomly to form new generation.

Chapter 2

Literature Review: Inspection of Multi Quality characteristics batch assembly Production

2.1. Outline of Chapter

In this chapter, literature on tolerance stack-up, Monte Carlo Simulation, quality control, inspection allocations in details. Also,literature on single quality characteristic batch assembly, proposed method for Multi quality characteristics batch assembly, Multi-stage production system, optimal inspection strategies are reviewed.

2.2. Introduction

Industrial scenario has always tried to produce the product which is cost-efficient and environment-friendly. Cost and quality are primary concern for any manufacturing industry. First, quality is a prime key to success of any industry. As discussed in previous chapter, company like Toyota has now become the brand name of quality as company continuously invest its money into quality products. In general, Somehow quality and cost of product both are acted as directly proportional. If quality is improved, total cost of that product is also increased as the inspection cost of that product is added into total cost. Now, discussing about inspection strategies, if company wants high-quality products without any defects, they need full inspection as well as tight the inspection limits. So they can send zero defect products to the customers. However, quality is increased by this method, but cost of that product is increased as tightening the inspection limits cause increase scrap and also increase rework cost. So, at the end, Total test cost is increased. So company requires balanced inspection strategy which can balance quality as well as cost.

In this thesis, multi quality characteristics for batch assembly line is calculated at each station. So at each station inspection allocation is decided and product is checked on different quality characteristics.

2.2.1. **Literature**

Many research articles have been published on the topic of the optimal allocation of the inspection policy for the multi-stage processes, i.e. on the decision where to place the inspection station in the process line.Lindsay and Bishop (1964) developed a concept of inspection in which total cost of inspection and scrap can be minimized. In that research, they put two inspection strategies: Full Inspection and No Inspection. They assume that full inspection is 100% efficient and they try to reduce that no defective product will reach to customer.VEATCH (2000) from Gordon College (Mathematics Department) developed inspection strategies and applied for thermal printer to find out the material which is going to take digital photograph is appropriate or not. Amit Verma (2004) implemented inspection strategies to find out defect in solder joint. They developed automated inspection system which can find the defect. Lin (2004) explored the inspection strategies to optimize the maintenance error which is result of production cost.Sung (2005) explored the requirement of time length for inspection model, expected cost for that length and optimize the strategies.E. Trovato (2010) explored the different strategies used to reduce rework cost or scrap. They compared different control methods and analyzed them for best strategy for particular problem.Chen (2013)explored optimal inspection frequency to reduce the rework cost as well as maintenance error and to get maximum profit. Muhammad Arsalan Farooqa (2017) explored different inspection strategies to minimize the cost as customer can get product at reasonable cost and higher in quality.Gianfranco Genta and Franceschini (2018) explained manufacturing assembly problems and designed new inspection strategies based on probability of defects on model.

Britney (1972) from the University of Western Ontario defined the quality control screening programs for the n-stage production process, and the criterion of the total expected cost was applied at all inspection stations, which constitutes the inspection cost, and the repair cost. White (1969) from MIT used the conclusions derived from Glenn and Albert's findings of using either full inspection or no inspection plan, developed a model of the shortest route which would help to determine the optimal location of the inspection station. Ballou and Pazer (1982) developed a model to determine the optimal location of the inspection station in an n-stage production systems, the model describes two types of inspector errors, the first being "predictable", in which the errors are known and constant, second being "erratic" which needs a variable which is random to describe the performance of the operator. Their results show that under some conditions, the level of predictable inspector errors significantly impacts the placement and the number of the inspection stations, and also it impacts the cost per item produced. Also their model was used by the management to traversing several policy options, like cost implications of increasing the quality vs the number of inspection stations. **Collins et al.** researched on the impact of the imperfect inspector that would be result of low quality of the goods from that inspection station and concluded that the impact could be more worse than it is expected. Gary D. Eppen and Jr (1974) gave a method for determining the location of the inspection station in a multistage production process, they use dynamic programming to prove that the optimal total cost function at every stage is linear and concave, so a time-sharing computer program is discussed. **Korytkowsi et al.** studied multiple product production manufacturing systems with in-line quality control and concluded that the quality control has an impact on the performance of the system because output is decreasing, and resource utilization is increasing.

2.3. Inspection Strategy

In this thesis, Multi quality characteristics batch assembly production cost is optimized by allocating different inspection allocation, which can reduce the cost. Full inspection and no inspection may understand by easy calculation, but when sample inspection is included in the allocation than it may be complex problem as sample Inspection depends on sample size and acceptance number. It is also not given the surety about the 100% accurate result as only samples are inspected. However, to reduce the cost, sample inspection plays a vital role. As allocating different inspection policy at each stage and run the simulation, then it gives the optimum solution. Inspection policy can be selected based on reducing the total inspection cost.

2.3.1. Example - Chen's Approach

As per Chen's approach of tolerance, there is four-part tolerance stack-up problem having Single quality characteristic. $X_{1,1}$ is the total length of assembly, so it is sum of four-part length.

$$x_{1,1} = x_1 + x_2 + x_3 + x_4 \tag{2.1}$$

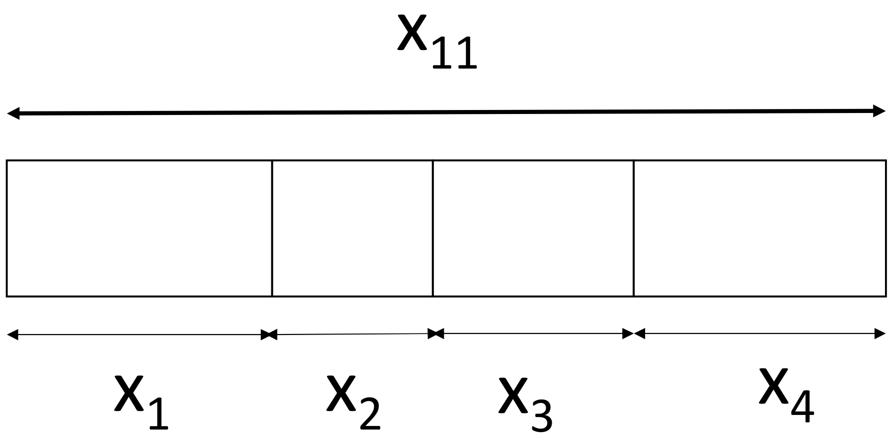

Figure 2.1: Four Part Assembly Tolerance Stack-up

Assuming normal distribution for this assembly. So two factors of normal distribution μ and σ are calculated for each part length.

$$\sigma_{1,1} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2} \tag{2.2}$$

where $\sigma_{1,1}$ = Standard deviation of overall length Let us consider,part 1 is inspected. so it has upper and lower inspection limits (u_1 and l_1).

$$l_1 = \mu_1 - \sigma_1 \cdot (t_1) \tag{2.3}$$

$$u_1 = \mu_1 + \sigma_1 \cdot (t_1) \tag{2.4}$$

where t = scalar (inspection parameter) During the inspection, cost of inspection C_i is added. If parts sent for rework then cost of rework is C_R added. If any delay or insufficient parts produced until the due date then penalty cost is C_p added. The decision tree for inspection policy is shown in fig. 2.2.

Total cost (C_t) is calculated by summation of each cost multiply by probability.

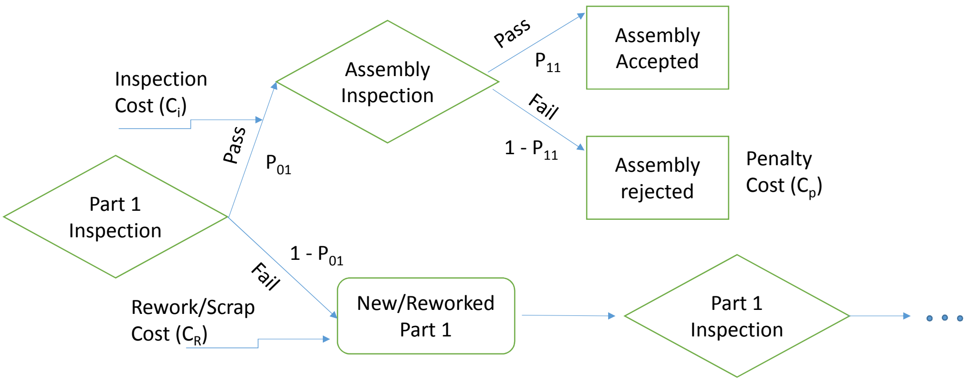

Figure 2.2: Process Chart for Quality Inspection
Chen (2013)

$$C_t = \frac{C_i}{P_{01}} + C_p \cdot (1 - P_{11}) + C_R \cdot \frac{(1 - P_{01})}{P_{01}} \tag{2.5}$$

Probabilities can be calculated as ;

$$P_{01} = \int_{L_{01}}^{U_{11}} pdf(x_{01}) \cdot dx_{01} \tag{2.6}$$

2.4. Concluding Remarks

As discussed above, to find the optimal solution for the single or multi quality characteristics problem for product for any assembly line, it requires algorithm and simulation matrix. Here, Monte Carlo Simulation is used to generate random numbers and find the solution and GA (Genetic Algorithm)is used to find the optimal solution so it will help to decide the inspection strategy for particular stage so total cost should be minimized. In this thesis, Penalty cost (if customer rejects the batch) is also included so overall total cost is optimized.

Chapter 3

Mathematical Model

3.1. Chapter outline

In this chapter, the inspection policies at different stations are allocated and costs are calculated by using mathematical equations. Discussion about MSPS in section 3.2, notations and calculation of costs are explained in section 3.3. Impact of quality characteristics on batch assembly line as well as unit production line are explained in section 3.4.

3.2. Production System

In the Multi-stage Production System (MSPS) is shown in fig. 3.1, number of stages are located (N). Each station consists of operation unit and inspection unit. Inspection unit is required because inspection policy allocated at the station. $N + 1$ station represents that product is reached to customer in this scenario. Penalty cost is also included, which is due to customers' refusal of batch. Different inspection policy is allocated at each station. The optimal policy is selected so total inspection policy cost (z) is reduced. The notation is described in the next section for better understanding.

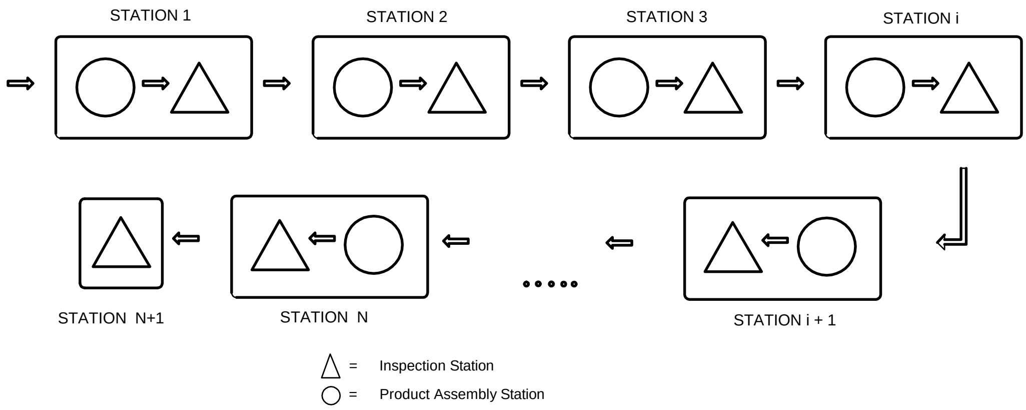

Figure 3.1: Multi Stage Production System

- N = No Inspection
- F = Full Inspection
- S = Sample Inspection

3.3. Notations :

Parameters:

K Batch size;

N Number of stages where stages are indexed by $i = 1, 2, \ldots, N + 1$. The index $N + 1$ stands for the customer site;

Q Number of quality characteristics indexed by $q = 1, 2, \ldots, Q$;

$Y_{i,q}$ A binary data equal to 1 if quality characteristic q is impacted in stage i, 0 otherwise;

L_q Lower specification limit after stage N for quality characteristic q;

U_q Upper specification limit after stage N for quality characteristic q;

T_i Unit test cost in stage i;

$R_{i,q}$ Unit rework cost in stage i for quality characteristic q;

P Unit penalty cost after stage N (for a product reaching the customer);

Policy Decision Variables (Independent Variables):

x_i Inspection option for stage i, i.e. $x_i \in \{F, N, S\}$;

$l_{i,q}$ Lower inspection limit in stage i for quality characteristic q.

$u_{i,q}$ Upper inspection limit in stage i for quality characteristic q.

s_i Sample size for stage i if sampling inspection is selected;

a_i Acceptance number for stage i for a sampling inspection;

Dependant Variables:

b_i Number of bad items in sample of stage i;

d_i Number of defective items after stage i;

t_i Total test cost in stage i;

r_i Total rework cost in stage i;

$p_{i,q}$ Fault occurrence rate in stage i for quality characteristic q;

z_1 Total inspection cost;

z_2 Total rework cost;

z_3 Total penalty cost;

z Total inspection policy or test cost;

3.3.1. Determination of total inspection policy cost (z) :

The objective is to minimize the total test cost explained in eq. 3.1.

Minimize:

$$z = z_1 + z_2 + z_3; \tag{3.1}$$

where:

$$z_1 = \sum_{i=1}^{N} t_i; \tag{3.2}$$

$$z_2 = \sum_{i=1}^{N} \sum_{q=1}^{Q} r_i; \tag{3.3}$$

$$z_3 = P \cdot d_{N+1}; \tag{3.4}$$

$$t_i = \begin{cases} T_i \cdot K & \forall i : (x_i = F) \vee ((x_i = S) \wedge (b_i > a_i)) \\ T_i \cdot s_i & \forall i : (x_i = S) \wedge (b_i \leq a_i) \\ 0 & \forall i : x_i = N \end{cases} \tag{3.5}$$

$$r_{i,q} = \begin{cases} R_{i,q} \cdot K \cdot p_{i,q} & \forall i : (Y_{i,q} = 1) \wedge [(x_i = F) \vee ((x_i = S) \wedge (b_i > a_i))] \\ R_{i,q} \cdot s_i \cdot p_{i,q} & \forall i : (Y_{i,q} = 1) \wedge [(x_i = S) \wedge (b_i \leq a_i)] \\ 0 & \forall i : x_i = N \end{cases} \tag{3.6}$$

Inspection policy can be selected to optimize the total inspection cost and to reduce the total overall cost. In the current Example, Consider sample inspection at station 1; full inspection at station 2; No inspection at station 3; sample inspection at station 4. Procedure is explained for both batch assembly production as well as unit production as per requirement.

3.4. Quality Characteristics

As discussed in the previous chapters, quality is our prime requirement. Inspection is carried out depends on how many factors would be checked during the process; what happens if any of the quality characteristics are not fulfilled by the manufacturer; what is impact of cost on quality. Quality can be checked for single characteristics or for multi quality characteristics. Now, understand both the criteria:

- **Single Quality Characteristic:**

  If inspection is allocated to check only one characteristic for all the station, it is called as single quality characteristic problem. For that, let us take an example of four stations in production unit.

  For single quality characteristic production process: Here, Inspection policy is allocated at each station. Inspection policy is assigned by X (which consist of values like inspection limits, type of inspection, sample size, acceptance number but it will be discussed later). For first stage, it is assigned as X_1. In the example, there are 4 stations so it is indicated as X_1, X_2, X_3, X_4 in sequence. Inspection policy is directly dependent on previous station inspection policy also. It is calculated as cumulative through the all stations and it is indicated as${X_i}^*$ where (i = number of stations). So, for this example it is calculated as

$$ {X_1}^* = X_1 \tag{3.7} $$

$$ {X_2}^* = X_2 + {X_1}^* \tag{3.8} $$

$$ {X_3}^* = X_3 + {X_2}^* \tag{3.9} $$

$$ {X_4}^* = X_4 + {X_3}^* \tag{3.10} $$

  Here in this example, Assuming normal distribution through all the stations.

- **Multi Quality Characteristics:**

  Multi Quality Characteristics consist of more than one characteristics are inspected at a station. So, if full inspection is assigned on a particular station with multi quality characteristics then all quality characteristics must be inspected and must have into inspection limits. If it fails in any particular characteristic then the cost of rework for only that characteristic is added in the system. Multi quality characteristics are assigned as q_i . and the index which shows the quality characteristics impacted on station is indicated as Y_i. The value of Y_i has remained in 0 and 1. (means, if quality characteristic is impacted then value is 1 and if not impacted than value is 0)

1. **Batch Assembly Production**

In the batch assembly production, let us consider batch size is 1000. The materials at each station as consider as raw material for that process station (because of considering assembly plant). From fig. 3.2, one batch contains 1000 raw materials are settled at station 1. At station 1, Inspection policy is allocated sample inspection. So it is required to decided the acceptance number and sample size. Let us consider sample size is 80, and Acceptance number is taken as 10 per sample (Means in the sample size = 10, and acceptance number = 2 then rejection parts must not be exceeded to 2. If rejected parts are more significant than 2 (acceptance number), Batch is rejected.) Now, start the inspection process at station 1, which is shown in fig 3.3. Let us inspected first 80 parts. Here in our case, Parts are always selected on random basis. (Note: We are taking first 80 because the parts are arranged already in raw randomly by default that's why we can take first 80 parts for sample inspection). Here acceptance number is 10, so if rejected parts are less than 10 then whole batch can be forwarded to the next station and the defected parts are going to rework. If rejected parts are more than acceptance number than batch is rejected. Now, inspected batch from station 1 is proceed for station 2. At this station, inspection policy is allocated as full inspection. So all the parts of Batch 1 are inspected at station 2, which is shown in fig. 3.4. Defected parts are going for rework. Meanwhile next batch will come at station 1.

After completing the inspection process at station 2, Batch 1 is moved to station 3, where no inspection policy is allocated, which is shown in fig 3.5. Means at station 3, none of the parts from batch is going to inspected. So the cost of inspection for this station is tends to zero. Meanwhile Batch 2 comes to station 2 for full inspection, and new batch (Batch 3) comes at station 1 for sample inspection, which is shown in fig. 3.5. After station 3, Batch 1 proceeds to station 4 for further process. On the station 4, Sample inspection is allocated. Here at station 4, criteria for the sample inspection is predefined like sample size is taken as 60, and acceptance number is taken as 5. (**Note:** If acceptance number is decided very tight (means low with compared to sample size),chances of rejection of whole batch is more. Although by applying this scenario (tight limit) may cause an increase of scrap and the overall cost is increased.However, quality is increased.)If condition of sample inspection is fulfilled, batch is going for dispatch and defected parts are going for rework, which is shown in fig. 3.6. New parts are replaced the defected parts. Meanwhile batch 2 is proceed at station 3; batch 3 is proceed at station 2, and new batch (batch 4) is proceed at station 1.

Now in batch assembly, Let us apply this inspection policy for 100 batches and find out the overall cost. After that, change the inspection policy (like at station 1, full inspection is allocated; at station 2, no inspection is allocated; at station 3, sample inspection is allocated; and at station 4, full

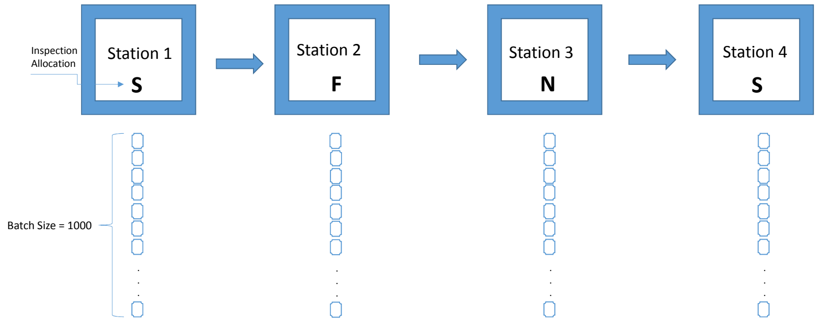

Figure 3.2: Batch Assembly Inspection Allocation

inspection is allocated) and then run simulation and find out the overall cost. By this way, manufacturer can optimize the cost and can establish the optimal inspection strategy for plant. (Note: Overall cost is dependent on inspection strategies. i.e., if applying full inspection is allocated at all station can increase the inspection cost but reduce the penalty cost from customer.)

2. **Unit Production**

This inspection allocation is also applicable to the unit production plant, which is shown in fig. 3.7. Let us consider the same inspection policy, which is considered in batch assembly layout. In unit production, for sample inspection, let's take first 5 samples for the inspection for first 100 units and considered acceptance number as 1. (Note: First 5 samples are also already arranged in randomly at station we can consider it as random sequence) At station 1, 1st part is inspected if it passes the inspection, it proceeds to station. Same like 1st part, another four parts are also inspected. Good parts are moved to the next station. If any part of five fails, it will send for rework. If more than 1 (acceptance number) part are found defected, inspection policy will be changed to full inspection. At station 2, full inspection is allocated. So all parts are inspected one by one at station 2, At

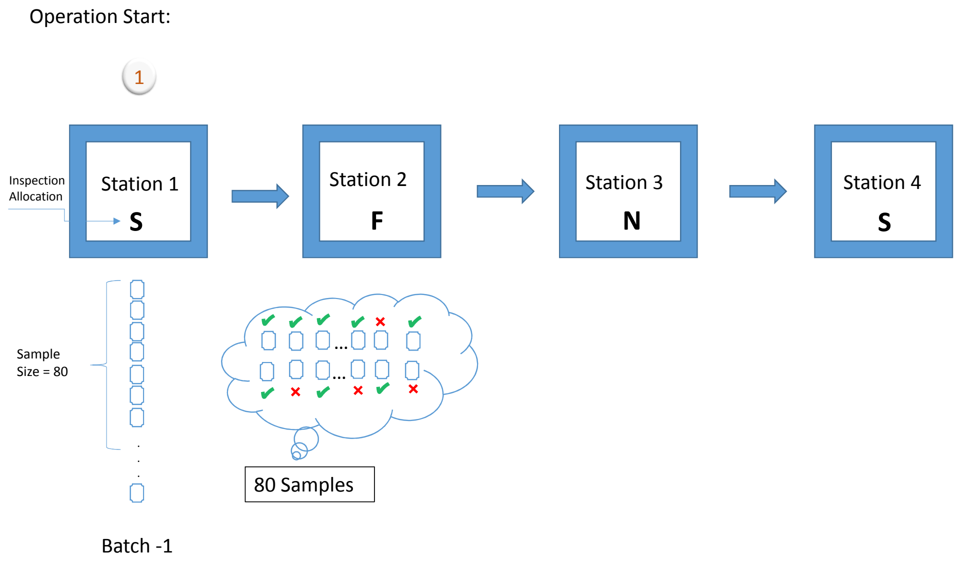

Figure 3.3: Batch Assembly Inspection Allocation - Station 1

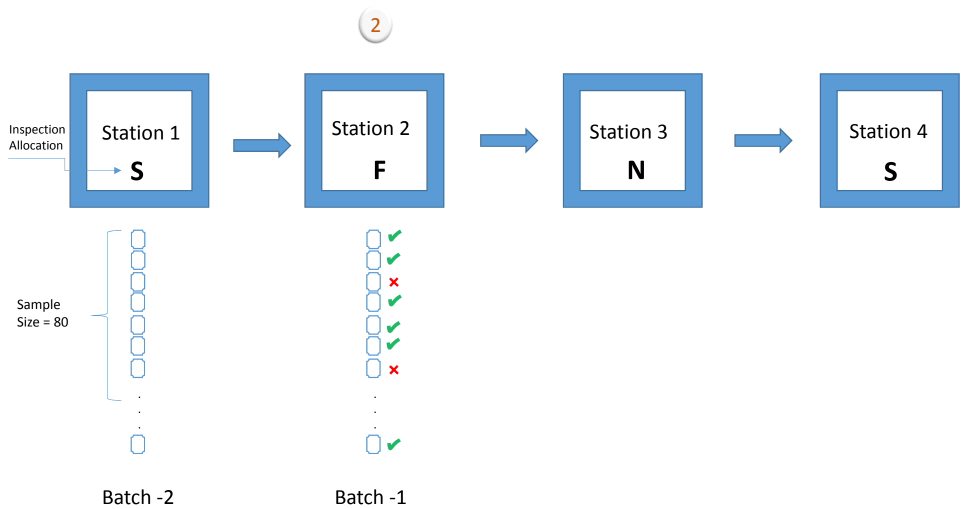

Figure 3.4: Batch Assembly Inspection Allocation - Station 2

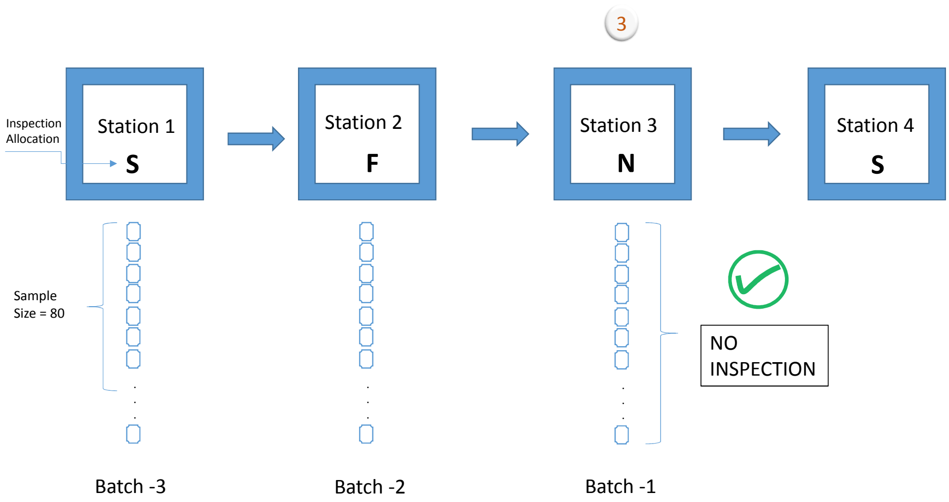

Figure 3.5: Batch Assembly Inspection Allocation - Station 3

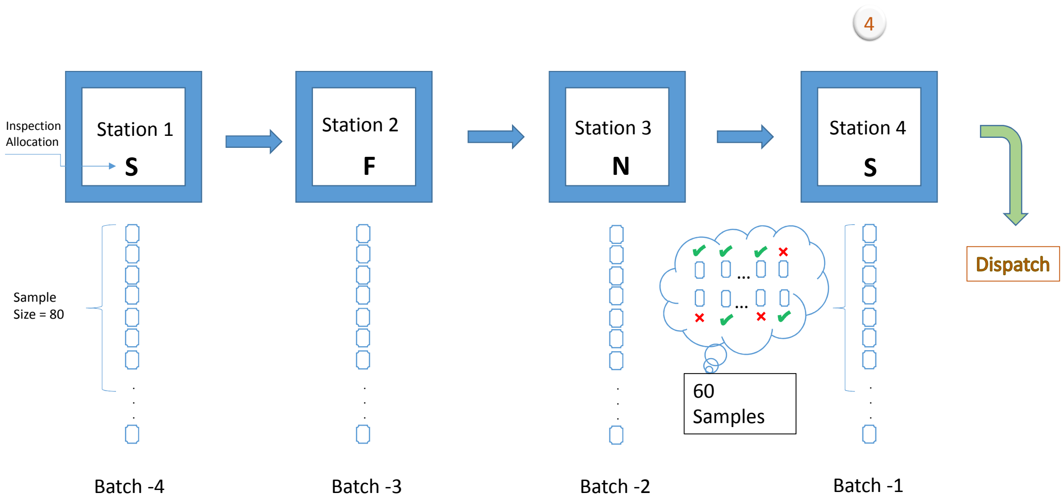

Figure 3.6: Batch Assembly Inspection Allocation - Station 4

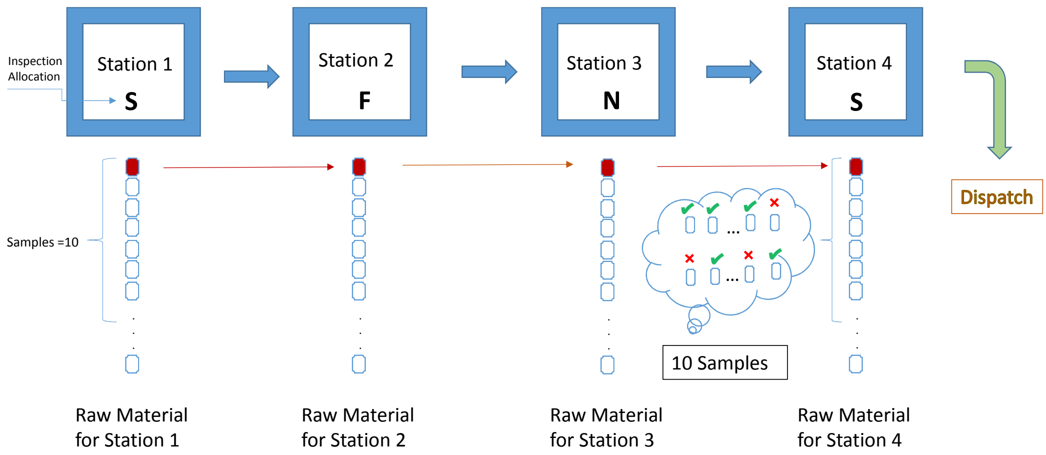

Figure 3.7: Unit Production - Inspection Allocation

station 3, no parts are inspected as no inspection is allocated. At station 4, same as station 1, sample inspection is allocated so random samples are checked and defected parts are sent for reworking. After station 4, parts are going to dispatch.

Chapter 4

Solution Procedure: Optimization Problem using Monte Carlo Simulation and Genetic Algorithm

4.1. Chapter Outline

In this chapter, Evolutionary algorithm is used to find the optimal solution for the problem. Monte Carlo Simulation and the Genetic Algorithm are used to solve the single and multi quality characteristics problem. In section 4.2, social representation of single quality characteristic problem is explained. In section 4.3, Reproduction process is explained for Genetic Algorithm. In section 4.4, the social representation of multi quality characteristics problem is explained.

4.2. Social representation of single quality characteristic multi-stage production system

Before understanding the problem of multi quality characteristics problem, let us first discuss about single quality characteristic problem. In every problem, decision variables are coded as genes, and all decision variables or genes are combined to make a chromosome. A solution of the algorithm is a representation of decision variables. In single quality characteristic problem Sofie Van Volsem (2007), reduce the cost of inspection by allocating inspection policy. For that it is required to determine the type of inspection allocation (x_i) , corresponding lower (l_i) and upper (u_i) inspection limits, sample size (s_i) and acceptance number (a_i) for all stages. By finding the optimal solution for stages $i = 1, 2 \cdots, n$ the finding solution for the set of

$$(x_1, x_2, \cdots, x_n; l_1, l_2, \cdots, l_n; u_1, u_2, \cdots, u_n; s_1, s_2, \cdots, s_n; a_1, a_2, \cdots, a_n) \quad (4.1)$$

that optimize the total inspection policy cost,

$$z(x_1, x_2, \cdots, x_n; l_1, l_2, \cdots, l_n; u_1, u_2, \cdots, u_n; s_1, s_2, \cdots, s_n; a_1, a_2, \cdots, a_n) \quad (4.2)$$

All the variables are chosen randomly in algorithm and try to optimize their values. For inspection limits l_i, u_i both should be trying to close to the mean value. So algorithm is trying to maximize the lower inspection limit (max l_i) so it can reach near to mean value and trying to minimize the upper inspection limit (min u_i)so it can be close to mean value which is shown in fig.4.1. By this way, quality of the product is increased. The algorithm will identify the inspection policy, sample size, and acceptance number (for sample inspection). For better understanding, let us take an example of single quality characteristic. In this

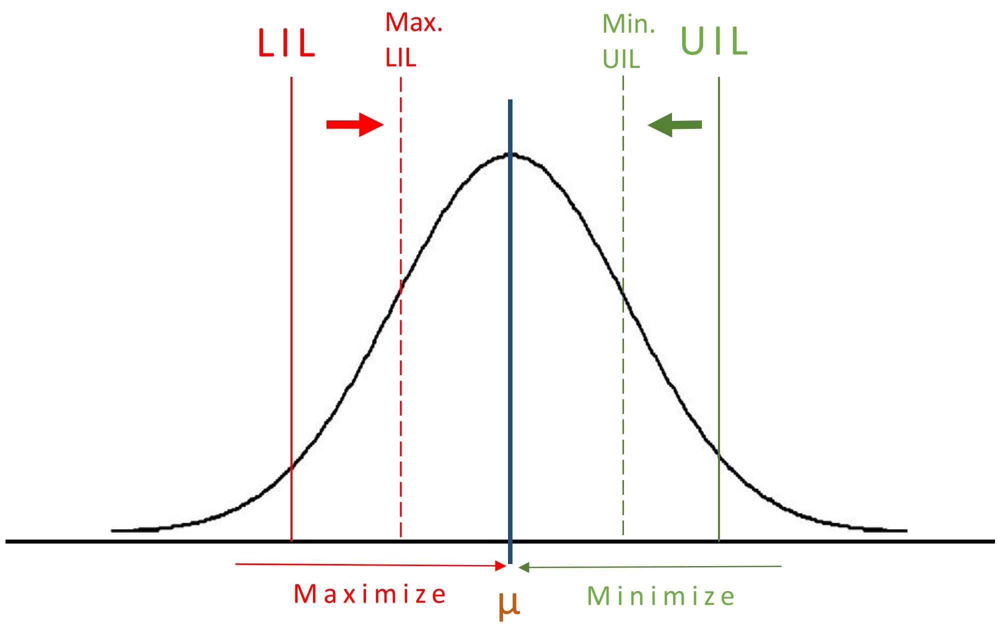

Figure 4.1: Max.Lower and Min. upper Inspection limit

example, there are three stages, and each stage has quality characteristic (q). Parameters are shown in table.

How algorithm works in our problem, explained in the following steps. These steps are same for the multi quality characteristics problem also.

- Read the data
- Read the parameters
- Initialize
- Evaluation
- Make new population
- Reproduction
- Make it old population

4.2.1. Read the data :

This is the first step of the GA. In any problem, data should be read by the program so to connect the data file with this algorithm is the most important thing. Reading the data consists of batch size, Number of quality characteristics impacted on stage, penalty cost, lower and upper specification limits, test costs, distribution data, mean and standard deviation values, rework costs, etc.

4.2.2. Read the Parameters:

Parameters can influence any problem. Parameter file may contain the data of population size, crossover and mutation probability, minimum and maximum values for the sample size and acceptance numbers, rejuvenation frequency and rejuvenation factor, maximum number of Monte Carlo simulation run etc. If there is no crossover probability then next generation is same copy of their parents. If mutation probability is maximum then chromosome is changed completely. So these probabilities are required to get optimized result.

4.2.3. Basic structure of problem

Gene Structure: starting with core variables which are going to find. Here, x = Inspection Policy, s = Sample size, a = Acceptance number of samples, l_i = lower inspection limit, u_i = upper inspection limit are constructed as genes. These genes make a chromosome. In tolerance stack-up problem, main concern is inspection limits.

After that, cost should be optimized, so making the structure for cost, which includes Total inspection cost, Total rework cost, Total penalty cost, and Total inspection policy cost. These costs are stored as an individual, so it is going to print in algorithm output.

In multistage production, the structure of stage consists all genes values and also it contains all the parameters which includes

- y = Quality characteristics impacted on stage or no. The value of y is 0 or 1 means if $y = 0$ then no quality characteristics impacted on stage and if y = 1 then quality characteristics is impacted on stage.

- $NQCE$ = Number of quality characteristics impacted on stage. $NQCE = 0, 1, 2, 3 \cdots Q$. For Single quality characteristic, its value is 0 or 1. However, for multi quality characteristics, it depends on how many quality characteristics are impacted at stage at same time.

- Distribution = If a value is one then Normal Distribution and if value is two, then uniform distribution is considered.

- Parameter 1 and Parameters 2: In this thesis, it is used to find the value of mean(μ) and standard deviation (σ). If distribution is Normal Distribution then

$$\mu = Parameter1 \tag{4.3}$$

$$\sigma = Parameter2 \tag{4.4}$$

  If there is an uniform distribution then

$$\mu = 0.5 * (Parameter1 + Parameter2) \tag{4.5}$$

$$\sigma = \sqrt{(Parameter2 - Parameter1)^2/12} \tag{4.6}$$

- Test Cost = Test cost is includes only if sample inspection and full inspection occurs at stage. Test cost is zero if no inspection policy is allocated at stage.

- Rework Cost = Rework cost is added when defected parts are found during sample or full inspection. Rework cost is zero if no inspection policy is allocated at stage. In single quality characteristic, rework cost is fixed but in multi quality characteristics it depend on parts fail in which quality characteristics so only that rework cost is added.

- Maximum lower inspection limit
- Minimum upper inspection limit

$$Maximum(l_{i,q}) = \mu - 0.5(\sigma) \tag{4.7}$$

$$Minimum(u_{i,q}) = \mu + 0.5(\sigma) \tag{4.8}$$

4.2.4. Initialization

Initialization is the process to start the genetic algorithm by making the population. This process is the same for the multi quality characteristics problem also. As discussed earlier, population consist of data of the genes. In this problem, genes are x, a, s, l_i, u_i which make one chromosome (so one chromosome contains 5 genes having values). Now for starting the process, values are randomly assigned to genes.

x is the inspection policy and it has 3 values (i.e 0 = No inspection, 1 = Sample inspection, 2 = Full inspection). so random numbers are lies between 0 to 2. s is sample size is selected randomly and in initial population, the value of s is in between max s to min s (i.e is already mention in parameters). Same as a acceptance number is chosen randomly and values lies between max a to min a.

If a quality characteristic is impacted on stage, values of lower inspection limit (l_i) and upper inspection limit (u_i) are chosen randomly in initial population. The equations to select these values are given below (for both single and multi quality characteristics):

$$l_{i,q} = \mu - (\sigma) * (0.5 + 2 * (randomvalue)) \tag{4.9}$$

$$u_{i,q} = \mu + (\sigma) * (0.5 + 2 * (randomvalue)) \tag{4.10}$$

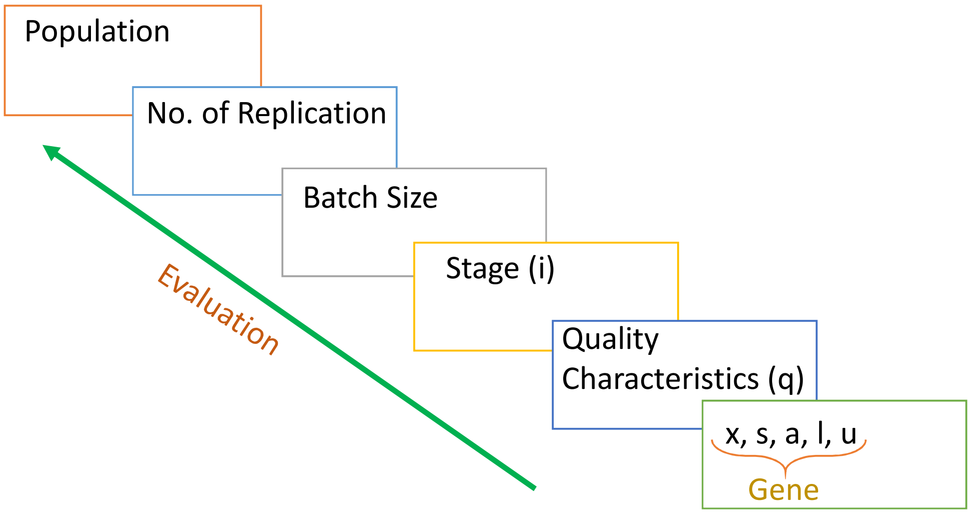

Figure 4.2: Evaluation steps

4.2.5. Evaluation - Monte Carlo Simulation

After generating the population in the initializing stage, it is time to find the best solution in terms of optimal total test cost. For that, in this thesis, Monte Carlo Simulation is used to find the best fitness individual. To generate random numbers (normal distribution or uniform distribution), Mersenne twister engine is used in this algorithm. Monte Carlo Simulation is run step by step, which is shown in fig.4.2.

For better understanding, let's take an example. Considering Population size = 50, Number of replication i.e Monte carlo simulation run $(NR) = 20$, Batch size$(K) = 100$, No. of stages $(i) = 3$, Number of quality characteristics impacted on stage = 1, Type of distribution = Normal Distribution From the above data, Simulation follow these steps.

- **Step 1 :** For the 1st replication of Monte Carlo Simulation,1st population is generated having the batch size and inspected at 3-stages having the standard value of x, a, s, l_1, u_1 (i.e generated randomly with in their range).

- **Step 2:** In the example, Normal distribution is selected, so the values of parts in a batch is distributed normally.(batch size K= 100, so 100 numbers are generated)

- **Step 3:** Now this batch is inspected at stage 1. There are three possibilities at stage 1.

  Step 3.0 : If at stage 1, the value of $x_1 = 0$ means No inspection policy is allocated at this stage. So Batch is passed to the next stage.

  Step 3.1 : If at stage 1, x_1 =1 means sample inspection is allocated then Sample size (s_1 and acceptance number a_1 are randomly selected between their range.

  Step 3.1.0 During the sample inspection if a part is passed the condition of acceptance (Dimension of part $> l_1$ and Dimension of part $< u_1$) then part is considered as quality part or good part otherwise considered as bad part of defected part.

  Step 3.1.1 : During the sample inspection if part is passed the condition ($b_1 \leq a_1$) then inspection cost for sample inspection (= T_1 * s_1) is added to the system.

  Step 3.1.2 : During an inspection, bad parts are found which are replaced by new part by changing the probability of standard deviation by ± 0.25. and rework cost(r_1) is added to the system. Defected parts (d_1) parts are counted in system

  Step 3.1.3 : If condition of acceptance number is violated (bad parts b_1 ¿ a_1), sample inspection is converted into full inspection. Total inspection cost ($t_1 = T_1$ * $(K - s_1)$ for full inspection for stage is calculated in the system.

During the Full inspection, if bad parts (b_1) are found then simulation repeat the step 3.1.2. and rework cost (r_1) is added to the system.

Step 3.2 : If at stage 1, $x_1 = 2$ means Full inspection is allocated. So all batch is inspected and follow the condition of acceptance (refer step 3.1.0) then total inspection cost for stage 1 (t_1) is added to the system. If any bad parts are detected, simulation repeats the step 3.1.2.

After stage 1, a batch is passed to stage 2 and then stage 3. all the time these steps are followed. After stage 3, Total penalty cost and total test cost for one replication are calculated by following equations:

- **Step 4:** Total penalty cost after stage 3, $z_3 = P * d/\text{NR}$;
- **Step 5:** Total test cost (z) for replication 1 = z_1 + z_2 + z_3

This simulation runs these steps for 20 replication (here $NR = 20$), and in every replication, Total inspection cost, total rework cost, total penalty cost, and total test cost are stored in the system. After completing 20 replication, the system stored the data of the final population (contain the value of total test inspection cost, total rework cost, total penalty cost, total test cost).

The calculation for 3-stage production system can be derived by,

$$X_1{}^* = X_1 * Y_1 \tag{4.11}$$

$$X_2{}^* = (X_2 * Y_2) + X_1{}^* \tag{4.12}$$

$$X_3{}^* = (X_3 * Y_3) + X_2{}^* \tag{4.13}$$

where,

X = containing data of (x, a, s, l_1, u_1)

Y = Quality characteristics impacted on stage or not. Value of Y = 0 or 1. If quality characteristics is impacted on stage then Y = 1 otherwise Y = 0.

4.3. Reproduction - Genetic Algorithm

After generating the population with their fitness value (by Evaluation process), the reproduction process is applied to find the optimal solution.

why reproduction is applied ?

In this thesis, we are trying to achieve the optimal solution for MSPS. After getting the values from the Evaluation process,it is necessary to find only the best fitness value for the next generation of replication. By selecting only the best fitness individuals from evaluation population, we can get an optimal solution because it is not necessary that we get only the best fitness values after the evaluation process, so reproduction is mandatory in our case.

Reproduction is divided into four-steps:

- Selection
- Crossover
- Mutation
- Making New Population

Reproduction is further explained in multi quality characteristics problem in section 4.4.

4.4. Social representation of multi quality characteristics multi stage production system

Multi quality characteristics problem is little bit different from the single quality characteristic problem. In single quality characteristic problem, only one quality characteristic is impacted at a time at each station. The part has been checked for only one quality characteristic. If part violates the condition, it goes to rework

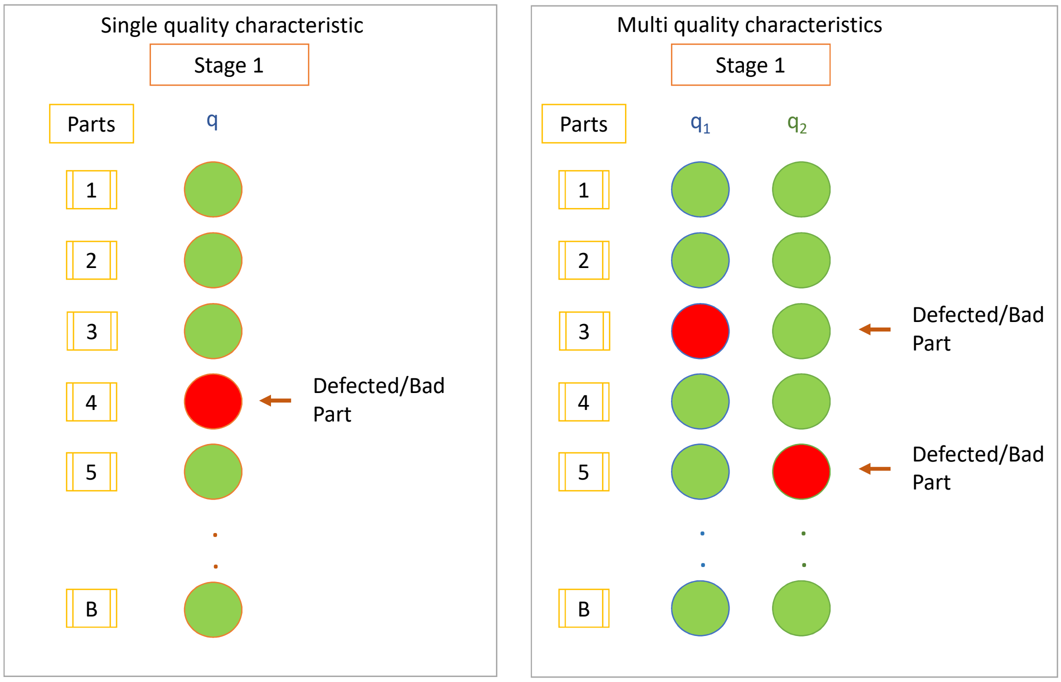

Figure 4.3: Comparison between Single and Multi quality characteristics

or scrap. However, in multi quality characteristics problem, parts have been inspected more than one quality characteristics at a time. So part fails in any of the quality characteristics; it is found to be a bad or defected part.

To find the optimal solution for multi quality characteristics problem is same as the single quality characteristic problem with some changes. The genes in multi quality characteristics problem can be defined as (x_i, a_i, s_i, $l_{i,q}$, $u_{i,q}$). Genes are described the decision variables. The changes are only in the genes of lower inspection limit and upper inspection limit because these variables are dependent on the characteristics. In the single quality characteristic problem, there is only one characteristic impacted, so at that time, there is no need to specify the number of characteristics in genes. However, our major concern is number of characteristics impacted on stage.

For better understanding, let us take a simple example of multi quality characteristics problem which is shown in fig.4.5.

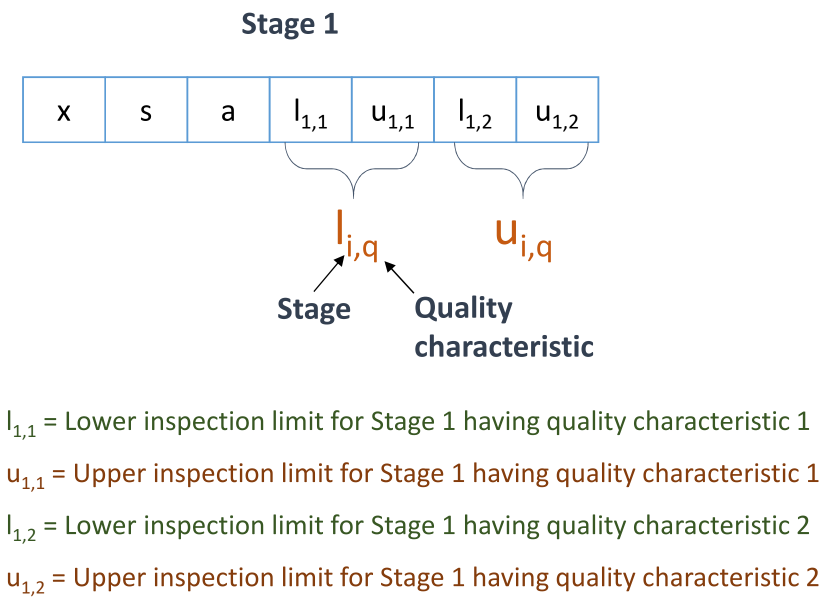

Figure 4.4: Gene structure in multi quality characteristics

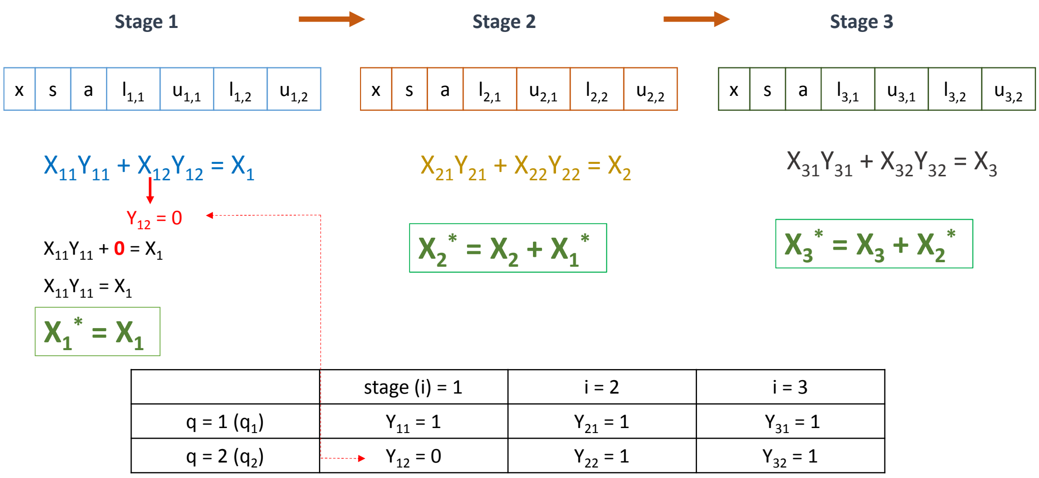

Figure 4.5: Multi quality characteristics Problem

In this problem, two quality characteristics are impacted on three stages. Quality characteristic (q_1) is impacted on stage 1 and stage 2. While quality characteristic (q_2) is impacted on stage 2 and 3. It is seen that in stage two quality characteristics are impacted. The solution representation steps are same as the single quality characteristic problem described in section 4.2. To understand the Monte Carlo Simulation (Evaluation step) for multi quality characteristics problem, Let's understand it by simple calculation which is shown in fig.4.5.

Calculation for n-stage production system, it is derived by,

$$X_n{}^* = (X_n) + X_{n-1}{}^* \tag{4.14}$$

In this thesis, one multi quality characteristics problem is taken as an experiment to find an optimal solution.

Table 4.1: Problem data

Parameters	values
Batch size (K)	200
No. of stages (i)	8
No. of quality characteristics (Q)	3
Penalty cost (P)	3000

Table 4.2: Quality characteristics impact at different stages

No.	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
q_1	1	1	1	0	0	0	0	0
q_2	0	1	1	1	1	1	0	0
q_3	0	0	0	0	1	1	1	1

The algorithm repeats the same steps, which is described in section 4.2. in which reading the data and reading the parameters have gone through same procedure as explained in section 4.2.1 and 4.2.2.

Stage	Quality characteristics	Distribution	Parameter 1	Parameter 2
1	q_1	Normal	10	0.1
2	q_1	Normal	10	0.2
	q_2	Normal	11	0.1
3	q_1	Uniform	8.5	11.5
	q_2	Uniform	13.5	14.5
4	q_2	Normal	9	0.1
5	q_2	Normal	10	0.1
	q_3	Normal	10	0.2
6	q_2	Normal	14	0.2
	q_3	Normal	12	0.1
7	q_3	Normal	10	0.5
8	q_3	Normal	16	0.8

Figure 4.6: Parameters for problem

Table 4.3: Test cost and Rework cost as per Quality characteristics

stages	Quality characteristics	Test cost	Rework cost
1	q_1	1	50
2	q_1, q_2	1	100, 50
3	q_1, q_2	2	200, 100
4	q_2	1	200
5	q_2 ,q_3	1	300, 100
6	q_2 ,q_3	1	400, 150
7	q_3	1	250
8	q_3	1	350

4.4.1. Initialization for multi quality characteristics:

As discussed in section 4.2.4, first population is generated randomly within the specific limits of genes. Here, genes or variables are (x_i, a_i, s_i, $l_{i,q}$, $u_{i,q}$) and these variables are randomly assigned.In this problem, 3-quality characteristics are impacted on stages. Mean, and standard deviation values are dependent on type of quality characteristics. Their values are taken as

If normal distribution,

$$\mu_{i,q} = [Parameter1]_{i,q} \tag{4.15}$$

$$\sigma_{i,q} = [Parameter2]_{i,q} \tag{4.16}$$

If there is an Uniform distribution then

$$\mu_{i,q} = 0.5 * [Parameter1 + Parameter2]_{i,q} \tag{4.17}$$

$$\sigma_{i,q} = [Parameter2 - Parameter1]_{i,q}/\sqrt{12} \tag{4.18}$$

The value of mean and standard deviation is incremental from stage to stage like

$$\mu^* = \mu_1 + \mu_2; \tag{4.19}$$

$$(\sigma^2)^* = \sigma^2{}_1 + \sigma^2{}_2; \tag{4.20}$$

x_i has 3 values (0,1 and 2). s_i and a_i are lies in between Maximum and minimum of their range. Now, only changes happen in values of $l_{i,q}$ and $u_{i,q}$ as they are dependent on quality characteristics which are derived by,

$$l_{i,q} = \mu_{i,q} - (\sigma)_{i,q} * (0.5 + 2 * (randomvalue)) \quad (4.21)$$

$$u_{i,q} = \mu_{i,q} + (\sigma_{i,q}) * (0.5 + 2 * (randomvalue)) \quad (4.22)$$

After generating initial population, next step is Evaluation (Monte carlo Simulation).

4.4.2. Evaluation for multi quality characteristics:

The evaluation process is described in section 4.2.5 for single quality characteristic. There are some changes for multi quality characteristics problem which will be explained step by step. As to generate random numbers, Mersenne twister engine is used in algorithm in this thesis. In the above example, Batch size K =200 , Number of stages (i) = 8, Number of quality characteristics impacted = 3, Number of MCS run = 50, Sample size (Max s , Min s) = (10, 5), Acceptance number (Max a , Min a) = (2, 1).

From the above data, Simulation follow these steps.

- **Step 1 :** For the 1st replication of Monte Carlo Simulation,1st population is generated having the batch size = 200 which is inspected at 8-stages having the standard value of x_i, a_i, s_i, $l_{i,q}$, $u_{i,q}$ (i.e generated randomly with in their range).For the 1st stage, Chromosome value is x_1, a_1, s_1, $l_{1,1}$, $u_{1,1}$, $l_{1,2}$, $u_{1,2}$, $l_{1,3}$, $u_{1,3}$. Now after reading the data, in the first stage only 1 (q_1) quality characteristic is impacted.

- **Step 2:** In the example, for the 1st stage, Normal distribution is selected, so the values (for parts) in a batch is distributed normally.

- **Step 3:** Now this batch is inspected at stage 1. There are three possibilities at stage 1.

  Let us assume at stage 1, No inspection policy is allocated. so batch is directly pass to next stage without inspection. (as single quality characteristic is impacted at stage 1 so this kind of problem is already discussed in section 4.2). Now batch one is passed to the next stage, i.e., stage 2. Now starting with step 3 for stage 2 having two quality characteristics q_1 and q_2.

  Step 3.0 : If at stage 2, the value of $x_2 = 0$ means No inspection policy is allocated at this stage. So Batch is passed to the next stage.

  Step 3.1 : If at stage 2, x_2 =1 means Sample inspection is allocated then Sample size (s_2 and acceptance number a_2 are randomly selected between their ranges.

  Step 3.1.0 During the sample inspection for multi quality characteristics problem, if part is passed the condition of acceptance (Dimension of part $>$ $l_{i,q}$ and Dimension of part $< u_{i,q}$) then part is considered as quality part or good part otherwise considered as bad part. In multi quality characteristics, if part violates any of the characteristics condition, it is considered as a bad part and send it for rework. Only that rework cost is added to system in which part is failed in quality characteristics. In this example, if one part is failed in condition of q_2, so rework cost($r_{i,q}$) to repair that part to satisfied the condition of q_2, rework cost of q_2 at stage 2 (= 50 , which is given in this example) is added in system.

  Step 3.1.1 : During the sample inspection, if part is passed the condition ($b_2 \leq a_2$) then inspection cost for sample inspection ($= T_2 * s_2$) is added to the system.

  Step 3.1.2 : During the inspection, bad parts are found which are replaced by new part by changing the probability of standard deviation by $\pm$ 0.25.

and rework cost($r_{2,2}$) is added to the system. Defected parts (d_2) parts are counted in system

Step 3.1.3 : If condition of acceptance number is violated (bad parts b_2 ¿ a_2), sample inspection is converted into full inspection. So Total inspection cost ($t_2 = T_2 * (K - s_2)$ for full inspection for stage is calculated in the system. During the full inspection, if bad parts (b_2) are found then simulation repeat the step 3.1.2.

Step 3.2 : If at stage 2, $x_2 = 2$ means full inspection is allocated. So all batch is inspected and follow the condition of acceptance (refer step 3.1.0) then total inspection cost for stage 2 ($t_2 = T_2 * K$) is added to the system. If any bad parts are found, simulation repeats the step 3.1.2. Moreover, the rework cost ($r_{2,q}$) is added to the system.

After stage 2, a batch is passed to stage 3 and then up to stage 8. all the time these steps are followed. After stage 8, Total penalty cost and total inspection policy cost for one replication are calculated by following equations:

- **Step 4:** Total penalty cost after stage 8, $z_3 = P * d$ / NR ;
- **Step 5:** Total inspection policy cost (z) for replication = $z_1 + z_2 + z_3$;

The calculation for this example,

$$X_1 = X_{1,1} * Y_{1,1} + X_{1,2} * Y_{1,2} + X_{1,3} * Y_{1,3} \tag{4.23}$$

In this example, Quality characteristic (q_2) and (q_3) are not impacted on stage 1. so the value of ($Y_{1,2}$) and $Y_{1,3}$) = 0 in this equation. Now, total value at stage 1 is expressed by

$$X_1 = X_{1,1} * Y_{1,1} + 0 + 0; X_1 = X_{1,1} * Y_{1,1}; \tag{4.24}$$

$$X_1{}^* = X_1 \tag{4.25}$$

same as it is derived for stage 2,

$$X_2 = X_{2,1} * Y_{2,1} + X_{2,2} * Y_{2,2} + X_{2,3} * Y_{2,3} \tag{4.26}$$

In this example, Quality characteristic $(q_1)$is not impacted on stage 2. so the value of $(Y_{2,3}) = 0$ in this equation. Now, total value at stage 2 is expressed by

$$X_2 = X_{2,1} * Y_{2,1} + X_{2,2} * Y_{2,2} + 0; X_2 = X_{2,1} * Y_{2,1} + X_{2,2} * Y_{2,2}; \tag{4.27}$$

$$X_2{}^* = X_2 + X_1{}^* \tag{4.28}$$

Same for stage 8, it can be derived by,

$$X_8{}^* = (X_8) + X_7{}^* \tag{4.29}$$

This simulation runs these steps for 50 replication (here $NR = 50$), and in every replication, Total inspection cost, total rework cost,defected parts are stored in the system. After completing 50 replication, the system stored the data of the final population (contain the value of total inspection cost, total rework cost, total penalty cost, total inspection policy cost). The best fitness value is stored in the system. If any new value finds more significant than the old value, algorithm set that new value as best fitness value. Algorithm take the total inspection policy cost or total test cost value as a best fitness value.

4.4.3. Selection

After getting the population having fitness value, it is time to choose only the best fitness value from the population. For that, many selection operators are used. In this thesis, Tournament selection is used to make new population. In tournament selection process, first of all tour size factor has to defined in the parameter list. It is essential to decide the span of tournament.

Let's consider the tour size factor = 0.1. So Tournament size is calculated by,

Tournament size = Tour size factor * Population size

Making the small size tournament can give the best individual for next population. The tournament has played between the random individuals of the population. The winners of tournament are selected as parents for the crossover process to generate new population. Here, total test cost is taken as fitness value so lower the total test cost value, consider as a higher the fitness value.

4.4.4. Crossover

After generating the matting pool having the best fitness individuals, crossover operator works on it and make the new offsprings. For generating new population or offsprings from parents,Crossover probability plays an essential role because without crossover probability new offsprings are exact copy of parents. The value of Crossover probability is already defined in the parameters. If crossover probability is taken as 100%, all new offsprings are made by crossover. This process is required having some crossover probability of making new offspring having some good part of their parents to survive in the next generation.

4.4.5. Mutation

After the crossover process, new offsprings are generated. The mutation is the process which slightly changes the value of new offsprings for better result. There are many mutation operators used for this process like flip mutation, bit string mutation, Non-uniform and uniform mutation, etc. In this thesis, we are using the flip mutation technique to change some of the value of new offspring. For

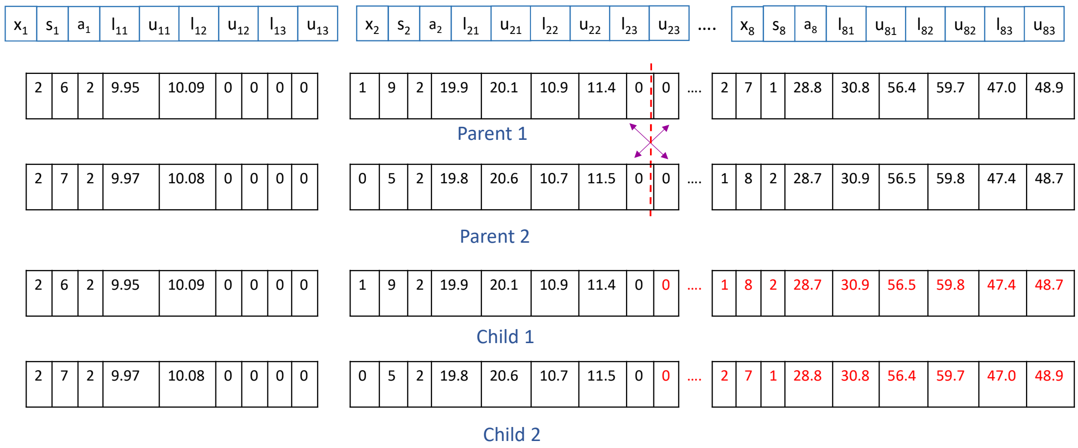

Figure 4.7: Crossover Process- GA

that, mutation probability plays an important role like crossover probability. If mutation probability is 0%, there is nothing to change in new offspring. If mutation probability is 100%, all values of chromosome are changed.But it is not good idea to change all the values of new offspring. Here in our case, in chromosome, there are 5 genes (x_i, a_i, s_i, $l_{i,q}$, $u_{i,q}$). So mutation applies to each genes having mutation probability. If random value of gene is lower than mutation probability, value of gene is flipped in certain amount.

x-Mutation: When the mutation process is applied on the x_i gene having the value $x_i = 0$ (No inspection) and the random value generated is less than the mutation probability then the value of gene is changed either 1 (sample inspection) or 2 (Full inspection) having 50% chances.

If x_i gene having the value $x_i = 1$ (Sample inspection) and the random value generated is less than the mutation probability, then value of gene is changed either 0 (No inspection) or 2 (Full inspection) having 50% chances.

If x_i gene having the value $x_i = 2$ (Full inspection) and the random value generated is less than the mutation probability, then value of gene is changed either 1 (sample inspection) or 0 (No inspection) having 50% chances.

L-Mutation:

Mutation operation is applied to the lower inspection limit gene also. Lower inspection limit is dependent on the quality characteristics. If the quality characteristics applied on the stage and the random value generated is less than the mutation probability then amount of lower inspection limit increase or decrease having 50% chances. The value of step amount increment or decrement is given by:

$$StepAmount = \sigma_{i,q}(0.1 + 0.2 * (randomvalue)) \tag{4.30}$$

If the step amount is increased, lower inspection limit is going closer to the mean value and that value stores in system.

U-Mutation:

Mutation operation is applied to the upper inspection limit gene also. Upper inspection limit is dependent on the quality characteristics If the quality characteristics applied on the stage and the random value generated is less than the mutation probability then amount of upper inspection limit increase or decrease having 50% chances. The value of step amount increment or decrement is given by eq.4.30. If step amount is decreased, upper inspection limit is going closer to the mean value and that value stores in system.

s-Mutation:

Mutation operation is applied to the sample size s_i gene. The random value generated is less than the mutation probability then value of gene is increased 1 or decrease 1 having 50% chances.

a-Mutation:

Mutation operation is applied to the acceptance number a_i gene. The random value generated is less than the mutation probability then value of gene is increased 1 or decrease 1 having 50% chances.

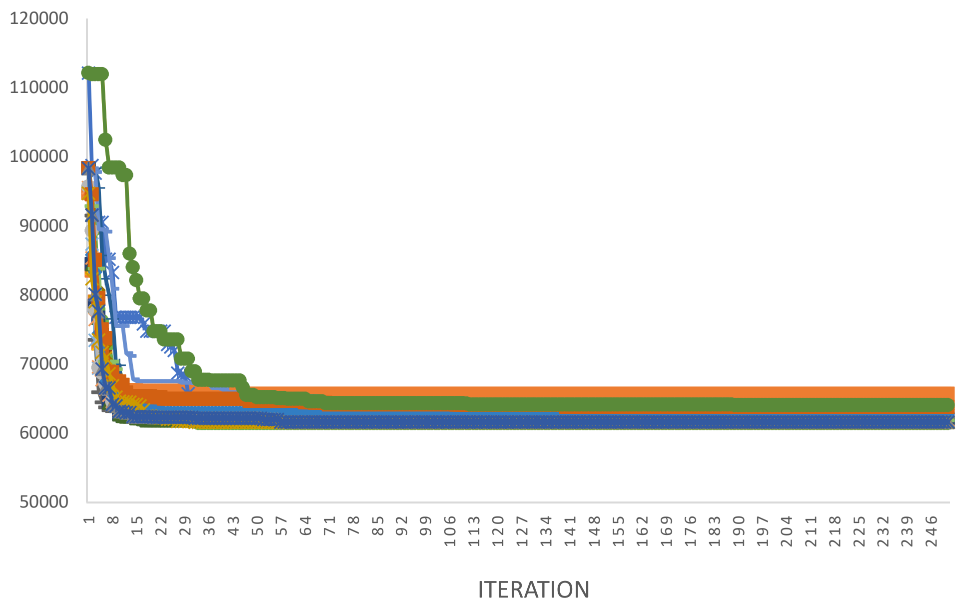

Figure 4.8: z (Total inspection policy cost) - Function of generation - for 25 replication of experiment problem

4.4.6. Make old population

After Mutation operation, the new population is generated. Moreover, this population is replaced the old population, which is generated in the initialization process. This cycle runs continuously and find optimal solution.

After reaching the Maximum number of generation, algorithm finds the optimal value for problem. For this experimental problem, maximum number of generation = 1000. After reaching this generation, optimal solution is found, which is given in Appendix A.

In the GA, optimal solution is dependent on the parameters. So it is possible to get different optimal solution by changing the parameters. Genetic algorithm runs simultaneously for same parameter of problem.

It is seen from graph that average value of total inspection policy cost has no such difference. It is lying in between (55000 to 66000).

It is also shown from fig.4.8, staring generations have some variation in total inspection policy cost. But after approximately 250 generation, total inspection policy cost are settled down near optimal value. [1]

[1] all costs will be considered in Canadian dollars throughout the thesis

Chapter 5

Numerical Examples

5.1. Comparison between different cases

In the experiment problem, different parameters will affect on the total test cost. For better understanding, the values of a parameter are changed one by one, and what will it affect on the result. In this thesis, there are different cases in which values are changed in experimental problem. Penalty cost, specification limits, test cost, rework cost, parameter 2 (which is mainly described the value of standard deviation) are effected factors to find the optimal solution. Batch size, number of stages, and number of characteristics have remained same in all cases, which is shown in table 5.1.

5.1.1. Case 1 - Penalty cost

As shown in table 5.2, penalty cost in the experiment data is high compare to this value. From this case, it can be seen that what is the optimal solution, if penalty cost is too low. By getting the solution for this case, the total test cost is decreased (z = 46054 approximately for this case) compare to experiment problem of multi quality characteristics. However, total inspection cost (z_1 =

410) and rework cost ($z_2 = 37900$) is decreased and total penalty cost ($z_3 = 7632$) is increased. It is obvious reason for these changes. Due to the low cost of penalty, algorithm s trying to allocate no inspection and sample inspection.

Table 5.1: Fixed Parameters

Parameters	values
Batch size (K)	200
No. of stages (i)	8
No. of quality characteristics (Q)	3

Table 5.2: Penalty cost comparison

Case study Problem	Experiment data
100	3000

5.1.2. Case 2 - Inspection cost

As shown in table 5.3, Test cost in the experiment data is low compared to these values. From this case, it can be seen that what is the optimal solution, if inspection cost is increased. High inspection cost is also a sign for high-quality products requirement. By getting the solution for this case, the total test cost is increased ($z = 64088$ approximately for this case) compare to experiment problem of multi quality characteristics. However, total penalty cost ($z_3 = 0$) and rework cost ($z_2 = 62470$) is decreased and total inspection cost is increased ($z_1 = 1618$).

5.1.3. Case 3 - Rework cost

As shown in table 5.4, Rework cost in the experiment data is high compare to these values. From this case, it can be seen that what is the optimal solution, if rework cost is decreased. Low rework cost is good for full inspection allocation.

Table 5.3: Test cost comparison

stages	Case study Problem	Experiment data
1	3	1
2	1	1
3	4	2
4	3	1
5	5	1
6	3	1
7	4	1
8	4	1

By getting the solution for this case, the total test cost is decreased ($z = 10564$ approximately for this case) compare to experiment problem of multi quality characteristics. However, total penalty cost ($z_3 = 0$) and rework cost ($z_2 = 9964.8$) is decreased and total inspection cost is increased ($z_1 = 600$).

Table 5.4: Rework cost comparison

stages	Quality characteristics	Experiment data	Case study problem
1	q_1	50	10
2	q_1, q_2	100, 50	20, 10
3	q_1, q_2	200, 100	20, 20
4	q_2	200	40
5	q_2 ,q_3	300, 100	60, 10
6	q_2 ,q_3	400, 150	80, 20
7	q_3	250	40
8	q_3	350	60

5.1.4. Case 4 - Parameter 2 and specification limits

As discussed in chapter 4, Parameter 1 and Parameter 2 are active factors which are required to find the mean and standard deviation value. In this case, changing the value of parameter 2 will affect the value of mean and standard deviation. In the data, values of parameter 2 are reduced, which mostly affects the standard deviation (the rejection of parts are increased if value of standard deviation is decreased). Same as the specification limits are affected. Here, in this case, lower and upper specification limits are too close to each other (means high accurate part is required having narrow tolerance) that leads to high-quality product but increase the scrap or rework cost.

Table 5.5: Parameter 2 - comparison

stages	Quality characteristics	Experiment data	Case study problem
1	q_1	0.1	0.051
2	q_1, q_2	0.2, 0.1	0.02, 0.01
3	q_1, q_2	11.5, 14.5	8.6, 13.6
4	q_2	0.1	0.01
5	q_2 ,q_3	0.1, 0.2	0.1, 0.2
6	q_2 ,q_3	0.2, 0.1	0.01, 0.02
7	q_3	0.5	0.05
8	q_3	0.8	0.08

Table 5.6: Problem data limits (lower and upper specification limits) comparison

Quality Characteristics	Case study problem	Experiment Problem
q_1	28.3, 28.8	29, 31
q_2	57.2, 58.1	56, 60
q_3	47.8, 48.3	47, 49

Now, replacing all the case study data with the value of experiment data, and then problem is simulated with Genetic algorithm for better understanding. After the simulation, optimize solution is generated having the value of Total inspection cost ($z_1 = 1800$), Total rework cost ($z_2 = 8918$), Total penalty cost ($z_3 = 646$), Total test cost ($z = 11364$).

Chapter 6

Discussion and Conclusion

The result of the experiment problem which is shown in fig. 6.3 is explained that total penalty cost is zero means the quality of the product is increased. Also, from the result, it concludes that no inspection policy is allocated more to reduce the inspection cost. At stage 1,2,5,6 and 7 has been allocated for no inspection strategy. Subsequently, sample inspection is allocated at stage 4, having an optimized sample size and respected acceptance number. Full inspection is allocated at stage 3 and 8. In the result, lower and upper inspection limits are optimized in that way to reduce the penalty cost and increase the quality. GA creates a solution with optimal total inspection policy cost of multi quality characteristics problem.

[Note: The experiment's result is generated by GA based on random numbers so that it may vary at every time due to randomness. In this thesis, after 1000 generation, optimal result is generated.]

Here one comparison is made between experiment problem and case study. Both are similar in comparison of stages and the number of quality characteristics impacted at each stage. The parameters, penalty cost, rework cost, and test cost is different from each other. The result of cost comparison and inspection policy allocation at each stage is shown in fig. 6.1. As in example 2, sample inspection is not allocated to any stage (shown in fig. 6.2) means sample size and acceptance number criteria does not match so whenever sample inspection allocated at stage and it fails in that criteria and then algorithm adopt full inspection. In example, test cost is high and penalty cost is low so algorithm try to optimize the total inspection policy cost. In the experiment problem, penalty cost is high, so algorithm try to minimize it hence minimize total inspection policy cost.

Cost	Experiment Problem	Case Study Problem
TIC (z_1)	605	1800
TRC (z_2)	59120	8918.6
TPC (z_3)	0	646
TTC (z)	59725	11364.6

Figure 6.1: Comparison of cost

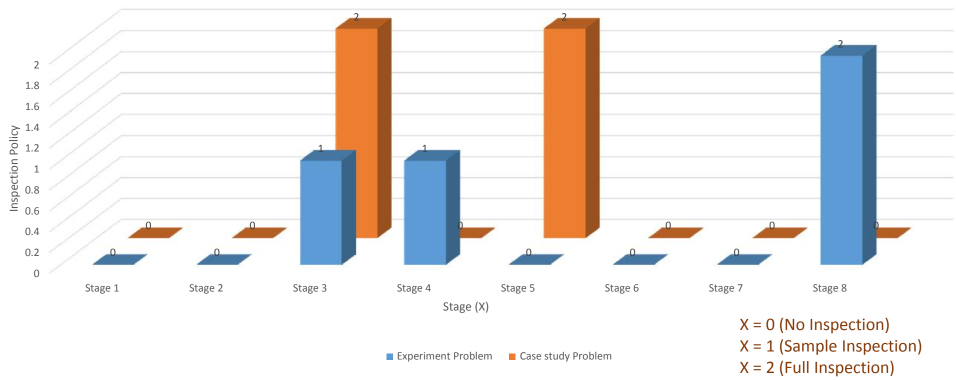

Figure 6.2: Comparison of inspection policy allocation

```
                                    GASolution
***********************************
Optimal Inspection Decisions:
***********************************

Stage-1
X = 0

Stage-2
X = 0

Stage-3
X = 2
q = 1   LIL = 29.5098   UIL = 30.9843
q = 2   LIL = 24.6605   UIL = 25.2885
S = 8   T = 1

Stage-4
X = 1
q = 2   LIL = 33.5884   UIL = 34.4623
S = 5   T = 2

Stage-5
X = 0

Stage-6
X = 0

Stage-7
X = 0

Stage-8
X = 2
q = 3   LIL = 47.0663   UIL = 48.9547
S = 10  T = 1
***********************************


***********************************
The corresponding cost values:
***********************************
TIC = 605
TRC = 59120
TPC = 0
TTC = 59725
***********************************
```

Figure 6.3: GA solution generated for experimental problem

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